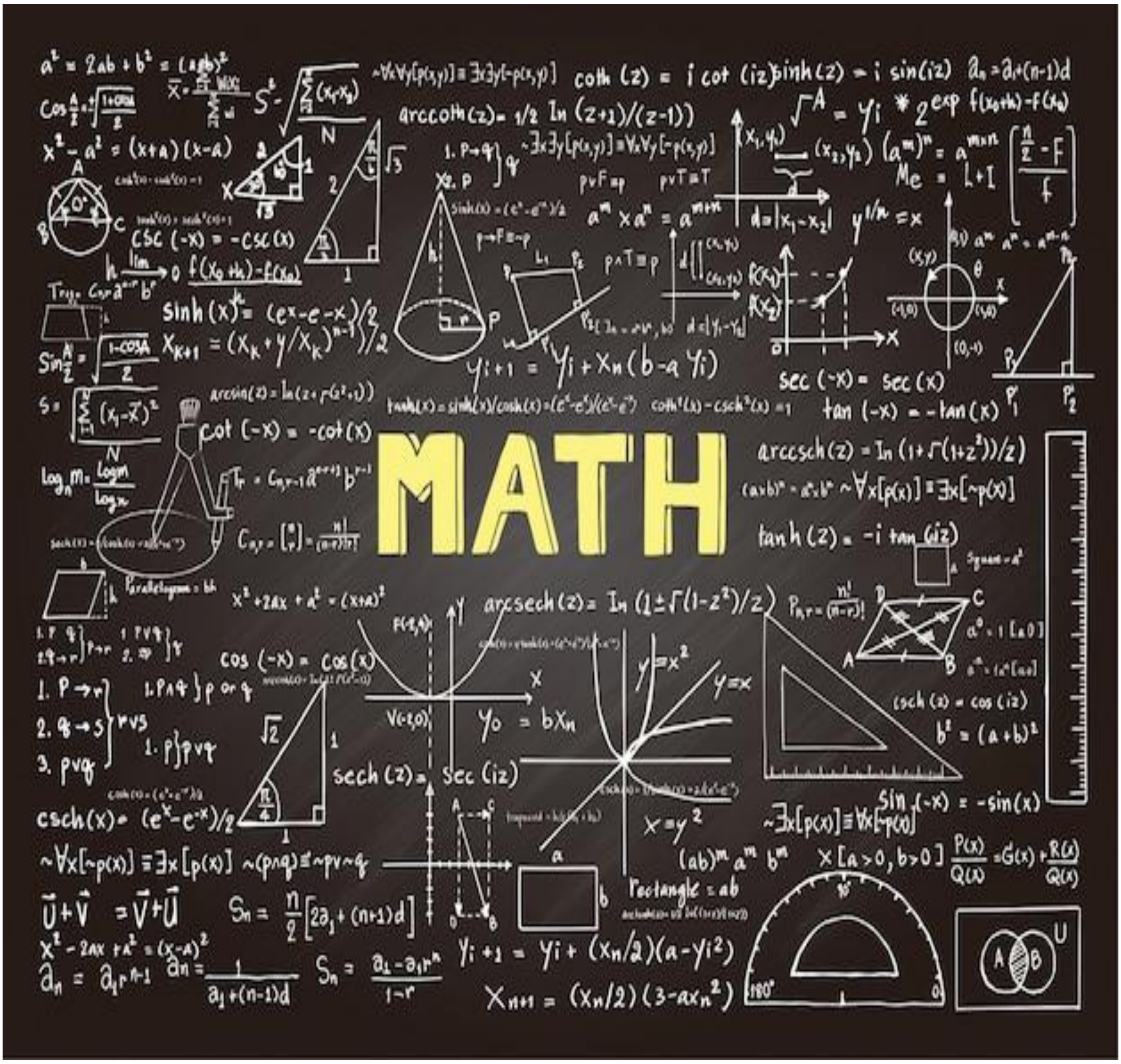
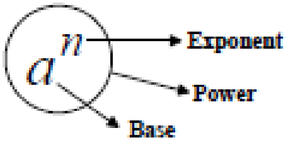


EXPONENTS
GRADE 10

CHAPTER 2 EXPONENTS

REVISION OF THE EXPONENTIAL LAWS AND DEFINITIONS

In earlier grades you were introduced to the expression
$a^n = a \times a \times a \times a \times \ldots$ (n factors)
where a represents the base, n the exponent and a^n the power.



Here is a summary and examples of the exponential laws and definitions. The bases are positive and the exponents are integers.

LAWS		EXAMPLES	
		Bases are variables	Bases are numerical
1.	$a^m . a^n = a^{m+n}$	$2x^4 . 3x = 2 . 3 . x^4 . x^1 = 6x^5$	$3^3 . 3^3 . 3^3 = 3^9 = 19\ 683$
2(a)	$\frac{a^m}{a^n} = a^{m-n} \quad (m > n)$	$\frac{y^7}{y^4} = y^3$	$\frac{6^6}{6^4} = 6^2 = 36$
2(b)	$\frac{a^m}{a^n} = \frac{1}{a^{n-m}} \quad (m < n)$	$\frac{y^4}{y^7} = \frac{1}{y^3}$	$\frac{6^4}{6^6} = \frac{1}{6^2} = \frac{1}{36}$
3.	$(a^m)^n = a^{m \times n}$	$(p^3)^2 = p^{3 \times 2} = p^6$	$(5^5)^5 = 5^{5 \times 5} = 5^{25}$
4.	$(ab)^m = a^m b^m$	$(4x^3 y)^2 = 4^2 x^{3 \times 2} y^2 = 16x^6 y^2$	$(2^6 . 3^8)^3 = 2^{18} . 3^{24}$
5.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{8d}{12e}\right)^3 = \left(\frac{2d}{3e}\right)^3 = \frac{8d^3}{27e^3}$	$\left(\frac{2^2}{5}\right)^3 = \frac{2^6}{5^3} = \frac{64}{125}$

DEFINITIONS		EXAMPLES	
		Bases are variables	Bases are numerical
1.	$a^0 = 1$	$5x^0 + (2x)^0 = 5 \times 1 + 1 = 6$	$6 . 3^0 = 6 \times 1 = 6$
2.	$x^{-n} = \frac{1}{x^n}$	$y^{-7} = \frac{1}{y^7}$	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
3.	$ax^{-n} = \frac{a}{x^n}$	$3x^{-4} = \frac{3}{x^4}$	$4.2^{-3} = 4.\frac{1}{2^3} = \frac{4}{2^3} = \frac{4}{8} = \frac{1}{2}$
4.	$(ax)^{-n} = \frac{1}{(ax)^n}$	$(2x)^{-3} = \frac{1}{(2x)^3} = \frac{1}{8x^3}$	$(4 \times 2)^{-2} = \frac{1}{(4 \times 2)^2} = \frac{1}{64}$
5.	$\frac{1}{x^{-n}} = x^n$	$\frac{1}{a^{-5}} = a^5$	$\frac{1}{5^{-3}} = 5^3 = 125$

6. $\frac{a}{x^{-n}} = ax^n$	$\frac{7}{y^{-6}} = 7y^6$	$\frac{2}{3^{-2}} = 2.3^2 = 2.9 = 18$
7. $\frac{1}{ax^{-n}} = \frac{x^n}{a}$	$\frac{1}{8p^{-9}} = \frac{p^9}{8}$	$\frac{1}{5.3^{-2}} = \frac{3^2}{5} = \frac{9}{5} = 1\frac{4}{5}$
8. $\frac{1}{(ax)^{-n}} = (ax)^n$	$\frac{1}{(2m)^{-4}} = (2m)^4 = 16m^4$	$\frac{1}{(3\times 2)^{-2}} = (3\times 2)^2 = 6^2 = 36$
9. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$	$\left(\frac{5x}{4y}\right)^{-2} = \left(\frac{4y}{5x}\right)^2 = \frac{16y^2}{25x^2}$	$\left(\frac{2}{7}\right)^{-2} = \left(\frac{7}{2}\right)^2 = \frac{49}{4} = 12\frac{1}{4}$
10. $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$	$\frac{a^{-5}.b^{-8}}{c^{-6}.d^2} = \frac{c^6}{a^5.b^8.d^2}$	$\frac{2^{-3}}{4.3^{-2}} = \frac{3^2}{4.2^3} = \frac{9}{4.8} = \frac{9}{32}$

OTHER RULES	EXAMPLES
1. $1^n = 1\times 1\times 1\times 1\times ....(n \text{ times}) = 1$	$1^{368} = 1$ $(80-79)^{1000} = 1^{1000} = 1$
2. $(-a)^n = a^n$ if n is even	$(-3)^4 = 3^4 = 81$ $(-1)^{2016} = 1^{2016} = 1$ $(-2x^3)^6 = (-2)^6.x^{18} = 2^6x^{18} = 64x^{18}$
3. $(-a)^n = -a^n$ if n is odd	$(-3)^5 = -3^5 = -243$ $(-1)^{2017} = -1^{2017} = -1$ $(-2x^3)^5 = (-2)^5.x^{15} = -2^5x^{15} = -32x^{15}$

EXERCISE 1 (REVISION OF GRADE 8 AND 9)

Simplify:

(a)
(1) $2p^3\times 8p^2$ (2) $6x^2y^3.6xy^4$ (3) $2^5.2^3.2$
(4) $9^4.9^8.3^2$ (5) $7^2.7^2.7$ (6) 7^2+7^2+7
(7) $5^5.5^5.5^5$ (8) 12.12^{12} (9) $3^{13}.3^{12}$
(10) $8^8.8^8$ (11) $3^2.2^2$ (12) 4.2^3
(13) $(3x^3y^5)^2$ (14) $2(2a^2)^3$ (15) $\left[3(2a^2b^3)\right]^2$
(16) $(2x^3)^3\times(3x^2)^2$ (17) $(2^5.6^7)^3$ (18) $3(3^3.2^3)^3$
(19) $\frac{x^5}{x^3}$ (20) $\frac{x^5}{x^9}$ (21) $\frac{8^3}{8}$
(22) $\frac{4^6}{4^{10}}$ (23) $\frac{-12a^{14}}{-18a^6}$ (24) $\frac{x^{40}y^{16}}{x^{36}y^{20}}$
(25) $\frac{-6a^3b^{10}c}{12a^8b^4c}$ (26) $\frac{(3x^2y^3)^2}{(3xy)(3xy^7)}$ (27) $\left(\frac{3a^3.2a^5}{12a^{10}b}\right)^2$

EXAMPLE 3

Simplify:

(a) $\dfrac{3^{x+1}+2.3^{x+2}}{7.3^x}$ (b) $\dfrac{9^x+3^x-2}{9^x-4}$ (challenge)

Solutions

(a) $\dfrac{3^{x+1}+2.3^{x+2}}{7.3^x}$

$=\dfrac{3^x.3^1+2.3^x.3^2}{7.3^x}$ [apply the rule $a^{m+n}=a^m.a^n$]

$=\dfrac{3^x(3^1+2.3^2)}{7.3^x}$ [factorise numerator]

$=\dfrac{3^x(3+2.9)}{7.3^x}$

$=\dfrac{3^x(21)}{7.3^x}$

$=3$ [simplify]

(b) $\dfrac{9^x+3^x-2}{9^x-4}$

$=\dfrac{(3^2)^x+3^x-2}{(3^2)^x-4}$ [write 9^x to base 3]

$=\dfrac{(3^x)^2+3^x-2}{(3^x)^2-4}$ [apply the rule $(a^m)^n=(a^n)^m$]

$=\dfrac{(3^x+2)(3^x-1)}{(3^x+2)(3^x-2)}$ [factorise the numerator and denominator]

$=\dfrac{(3^x-1)}{(3^x-2)}$ [simplify]

EXERCISE 4

(a) Simplify:

(1) $\dfrac{2^{x+2}+2^{x+3}}{12.2^x}$ (2) $\dfrac{3^{x+1}+3^{x+2}}{8.3^{x+1}}$ (3) $\dfrac{2^{x+2}-2^{x+1}}{2^x+2^{x+2}}$

(4) $\dfrac{5^{x+4}-5^{x+3}}{100.5^{x+1}}$ (5) $\dfrac{4^x+3.2^{2x+1}}{7.2^{2x+1}}$ (6) $\dfrac{(3^x)^2-9^{x-1}}{9^{x-1}}$

(7) $\dfrac{8^x.2^x+2.16^{x+1}}{11.2^{x+1}}$ (8) $\dfrac{12^x+4^x.3^{x+1}}{2^{2x+4}.3^x}$ (9) $\dfrac{2.3^x+3^{x-2}}{5.2^{x+1}-7.3^{x-1}}$