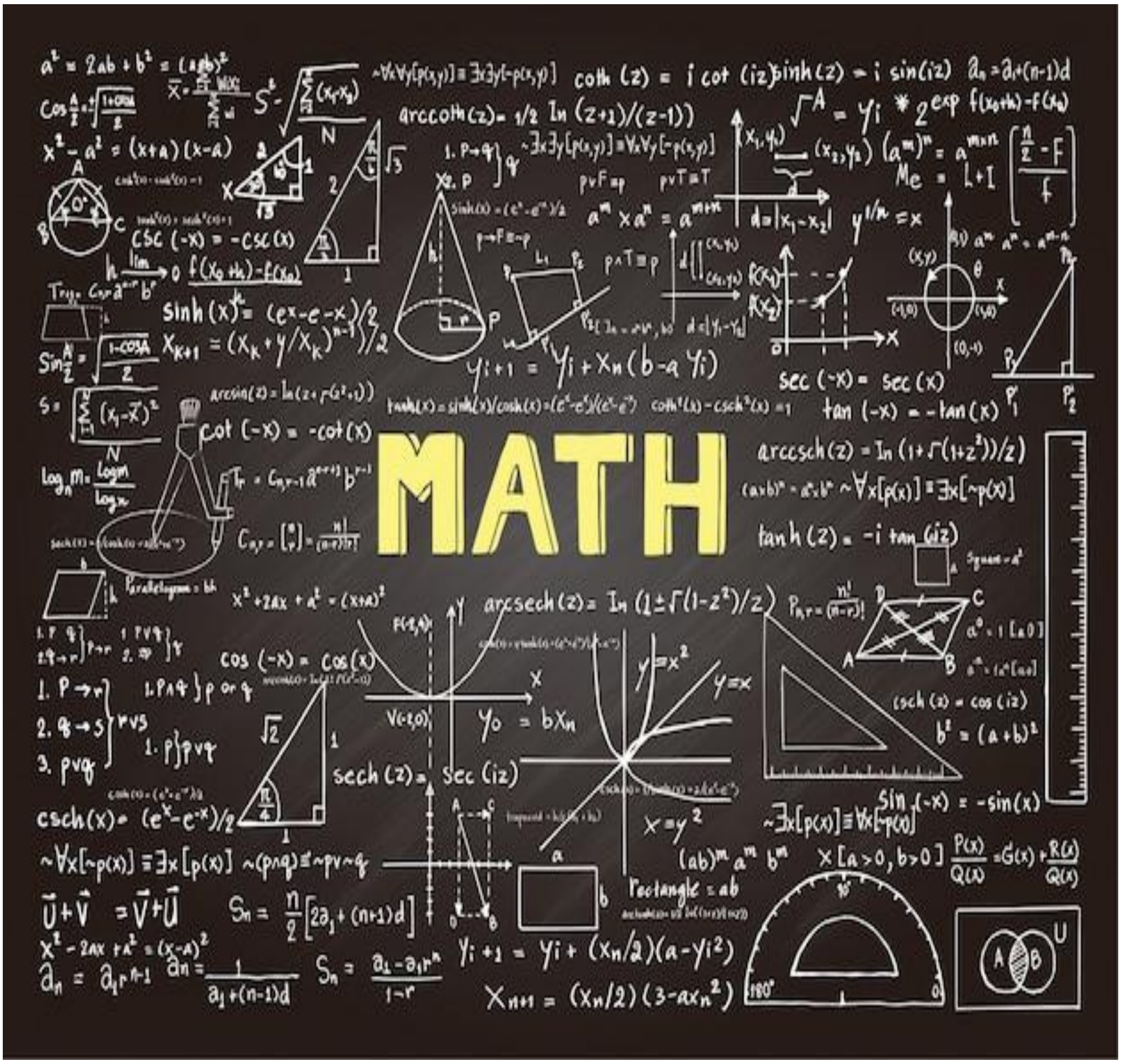
MATH

EXPONENTIAL EQUATIONS

In Grade 9 you learnt that in an exponential equation, the exponent is the unknown.

EXAMPLE 4

Solve the following equations:

(a) $4.25^{x+3}=4$ (b) $(0,5)^{x-1}=\left(\frac{1}{4}\right)^x$ (c) $9^x+3^{2x+1}=36$

Solutions

(a) $4.25^{x+3}=4$

$\therefore 25^{x+3}=1$	[divide both sides by 4]
$\therefore (5^2)^{x+3}=5^0$	[write 25 to base 5 and 1 as 5^0]
$\therefore 5^{2x+6}=5^0$	[multiply exponents]
$\therefore 2x+6=0$	[equate exponents]
$\therefore 2x=-6$	
$\therefore x=-3$	

(b) $(0,5)^{x-1}=\left(\frac{1}{4}\right)^x$

$\therefore \left(\frac{1}{2}\right)^{x-1}=\left(\frac{1}{2^2}\right)^x$	[write 0,5 as a fraction and 4 as 2^2]
$\therefore (2^{-1})^{x-1}=(2^{-2})^x$	[apply the definition $\frac{1}{a^n}=a^{-n}$]
$\therefore 2^{-x+1}=2^{-2x}$	[multiply exponents]
$\therefore -x+1=-2x$	[equate exponents]
$\therefore x=-1$	[solve]

(c) $9^x+3^{2x+1}=36$

$\therefore (3^2)^x+3^{2x}.3=36$	[write 9 to base 3]
$\therefore 3^{2x}+3^{2x}.3=36$	[apply the rule $a^{m+n}=a^m.a^n$]
$\therefore 3^{2x}(1+3)=36$	[factorise]
$\therefore 3^{2x}(4)=36$	[simplify]
$\therefore 3^{2x}=9$	[divide both sides by 4]
$\therefore 3^{2x}=3^2$	[write 9 to base 3]
$\therefore 2x=2$	[equate exponents]
$\therefore x=1$	[solve]

EXERCISE 5

(a) Solve the following equations:

(1)	$2^x = 1$	(2)	$3^x = 3$	(3)	$3^x = 27$	(4)	$4^x = 16$
(5)	$7^{4x} = 49$	(6)	$3.3^x = 243$	(7)	$2^{3x-1} = 64$	(8)	$121^{4x} = 11$
(9)	$3^{2(x-1)} = 81$	(10)	$5.5^{x-5} = 5$	(11)	$2.3^x = 162$	(12)	$8^x.2 = 128$
(13)	$3^x = \frac{1}{9}$	(14)	$4^x = \frac{1}{16}$	(15)	$5^x = \frac{1}{125}$	(16)	$\left(\frac{1}{2}\right)^x = 4$
(17)	$\left(\frac{1}{4}\right)^x = 16$	(18)	$\left(\frac{1}{3}\right)^x = \frac{1}{27}$	(19)	$3\left(\frac{1}{3}\right)^{x-1} = \frac{1}{3}$	(20)	$\frac{1}{8}.2^{2x} = 1$

(b) Solve the following equations:

(1)	$5.9^{x-1} = 5$	(2)	$7.49^{x+2} = 49$	(3)	$\frac{1}{49^x} = 343$
(4)	$5.125^{x+3} = \frac{1}{25}$	(5)	$\left(\frac{2}{3}\right)^{x-2} = \frac{8}{27}$	(6)	$(0,25)^x = 0,125$
(7)	$(0,2)^{x-2} = 0,04$	(8)	$0,4^x = 0,064$	(9)	$(3^{x+1})^3 = 9^{x-3}$
(10)	$4^x = 16^{x-1}$	(11)	$3^x.9^{x-1} = 81$	(12)	$(0,5)^{x-1} = 4^{-x}$
(13)	$81^{2x+1} = 27^{x-2}$	(14)	$8^{-x} = 2.4^{x-1}$	(15)	$4.2^x = (0,5)^{x-2}$

(c) Solve the following equations:

(1)	$2^{x+1} + 2^{x+2} = 24$	(2)	$5^{x+1} - 2.5^x = 75$	(3)	$3^x + 3^x + 3^x = 3^3$
(4)	$7^{x+1} + 14.7^x = 147$	(5)	$2.3^{2x+1} + 3.9^x = 243$		

EQUATIONS WITH RATIONAL EXPONENTS (FRACTIONS)

EXAMPLE 5

Solve the following equations:

(a) $x^{\frac{1}{3}} = 4$ (b) $x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 6 = 0$

Solutions

(a) $x^{\frac{1}{3}} = 4$

$\therefore (x^{\frac{1}{3}})^3 = 4^3$ [raise both sides to the reciprocal of the exponent]

$\therefore x = 81$

(b) $x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 6 = 0$

$\therefore (x^{\frac{1}{4}})^2 - 5x^{\frac{1}{4}} + 6 = 0$ [change the expression $x^{\frac{1}{2}}$ to $(x^{\frac{1}{4}})^2$]

$\therefore (x^{\frac{1}{4}} - 3)(x^{\frac{1}{4}} - 2) = 0$ [factorise the trinomial]

$\therefore x^{\frac{1}{4}} = 3$ or $x^{\frac{1}{4}} = 2$

$\therefore (x^{\frac{1}{4}})^4 = 3^4$ or $(x^{\frac{1}{4}})^4 = 2^4$

$\therefore x = 81$ or $x = 16$

EXERCISE 6

(a) Solve for x:

(1) $x^{\frac{1}{2}} = 5$ (2) $x^{\frac{1}{3}} = 2$ (3) $x^{\frac{1}{4}} = 1$

(4) $x^{\frac{1}{5}} = 2$ (5) $x^{\frac{2}{3}} = 4$ (6) $x^{\frac{3}{2}} = 27$

(7) $x^{\frac{5}{2}} = 32$ (8) $x^{\frac{7}{6}} = 128$ (9) $3x^5 = 729$

(b) Solve for x:

(1) $x^{\frac{1}{2}} - 7x^{\frac{1}{4}} + 12 = 0$ (2) $x^{\frac{1}{2}} - 8x^{\frac{1}{4}} + 16 = 0$ (3) $x^{\frac{1}{2}} - 8x^{\frac{1}{4}} + 7 = 0$

(4) $x^{\frac{1}{3}} - 6x^{\frac{1}{6}} + 8 = 0$ (5) $x^{\frac{1}{7}} - 3x^{\frac{1}{14}} + 2 = 0$ (6) $x - 9x^{\frac{1}{2}} + 18 = 0$

CONSOLIDATION AND EXTENSION EXERCISE

(a) Simplify without using a calculator:

(1) $2x^{-2} + (2x)^{-2}$ (2) $2x^0 + (2x)^0$ (3) $a^{-1} + b^{-1}$

(4) $(a+b)^{-1}$ (5) $(3011 - 3012)^{3013}$ (6) $(3x^4)^2 . 2(x^2)^4$

(7) $(3x^4)^2 + 2(x^2)^4$ (8) $(3x^3 + 3x^3)^3$ (9) $(-4x^2y)^2 . (-4x^2y)^3$

(10) $\left(\dfrac{12x^{-2}y^4}{18x^{-6}y^7}\right)^{-2}$ (11) $\dfrac{32^2 . 25^3}{100 . 8^4}$ (12) $\dfrac{8^{\frac{2}{3}} . (2^{-3})^{\frac{1}{3}}}{16^{\frac{1}{2}}}$

(13) $\dfrac{9^{x-1} . 24^{x+1}}{27^x . 8^x}$ (14) $\dfrac{2^{x+2} + 5 . 2^x + 2^x}{5 . 2^x}$ (15) $\dfrac{(x^4)^{-\frac{1}{8}} . (x^{\frac{1}{2}})^3 . (x^{\frac{1}{3}})^{-1}}{(x^{\frac{1}{3}})^2}$

(16) $\dfrac{121^x - 4}{11^x + 2}$ (17) $\dfrac{4^x - 3 . 2^{x+1} - 27}{2^x + 3}$ (18) $\dfrac{50^x - 10^x}{10^x - 2^x}$

(b) Solve for x:

(1) $4^x = 0{,}25$ (2) $9 . 27^{x-2} = 3$ (3) $16 . 4^{x+3} = 8^{x-2}$

(4) $2x^{\frac{3}{2}} = 250$ (5) $x^{\frac{1}{2}} - 8x^{\frac{1}{4}} + 15 = 0$ (6) $2^{3x+2} + 8^{x+1} = 48$

(c) Simplify:

(1) $4^x . 4^x . 4^x$ (2) $4^x + 4^x + 4^x$ (3) $4^{10} + 2 . 4^{10} + 3 . 4^{10}$

(d) (1) What is one half of 2^{24}?

(2) What is one sixth of 6^{36}?

(e) Show that:

(1) $8^8 + 4^{12} + 16^6 + 2^{24} = 2^{26}$

(2) 3^{30} is greater than 4^{20}

(f) If $a = 1 + 2^n$ and $b = 1 + 2^{-n}$, show that $b = \dfrac{a}{a-1}$.