1. Quadratic Function (Parabola)

A parabola looks like the mouth of a happy or sad face 😊 or ☹

The equation for a parabola is $y = ax^2 + q$

a determines how fat or thin the graph is – the larger the value of a is the thinner the graph. The closer to zero a is, the wider the graph.

The a value also tells whether the graph is happy or sad.

- If a is negative ($a < 0$) then the graph is sad.
- If a is positive ($a > 0$) then the graph is happy

The effect of q:

- Indicates whether the graph will move up or down
- Indicates the turning point of the function
- Represents the y-intercept
- Gives an indication of the Range
- Indicates whether the function has a maximum or minimum value

Turning Point

- The turning point happens when the graph changes from a negative to a positive gradient or from a positive to a negative gradient.
- In grade 10, the turning point is always the y-intercept coordinate.
- The turning point is the minimum or maximum point of the graph.
- The turning point is given by the coordinate (0; q).

Axes of Symmetry

- A parabola has a single axis of symmetry, which passes through the turning point of the graph.
- The axis of symmetry is a straight line that splits the graph exactly in two so that each half is a mirror image of the other half.

Practice Exercise 1

Sketch the following on a set of axis and answer the questions that follow.

1.1 $f(x) = -x^2 + 4$

a) Write down the range of f
b) Write down the turning point of f
c) Determine the value of x for which $f(x) > 0$
d) Write down the equation of h, where $h(x) = f(x) + 2$

1.2 $g(x) = -3x^2 + 27$

a) Write down the range of g

b) Write down the turning point of g

c) Determine the value of x for which $g(x) \leq 0$

d) Write down the equation of the symmetry line.

e) Write down the equation of h, where $h(x) = -g(x) - 2$

1.3 f$(x) = \frac{1}{2}x^2 - 1$

a) Write down the range of f

b) Write down the turning point of f

c) Determine the value of x for which $f(x) \geq 0$

d) Write down the range of h, where $h(x) = f(x) + 3$

1.4 $g(x) = -\frac{1}{3}x^2 + 2$

a) Write down the range of g

b) Write down the turning point of g

c) Determine the value of x for which $g(x) > 0$

d) Write down the equation of the symmetry line.

e) Write down the range of h,where $h(x) = -g(x) + 1$

2 Hyperbola

The equation of hyperbola is: $f(x) = \frac{a}{x} + q$

The hyperbola has two asymptotes **(electric fence)**;

- Horizontal asymptote; $y = q$
- Vertical asymptote; $x = 0$

Domain and Range

- Domain: all possible values of x but $x \neq 0$
- Range: all possible values of y: $y = q$
- N.B. We must exclude the asymptotes from the Domain and Range.

The effect of a

- Indicates if the graph is increasing or decreasing i.e. positive or negative
- Indicates the position of the curves on the Cartesian plane.

The effect of q

- Represents the horizontal asymptote
- Indicates when the graph will move up or down
- It is a restriction when finding the Range

Axes of Symmetry

- The hyperbola has two lines of symmetry
- One with the positive gradient; $y = x + q$
- With negative gradient; $y = -x + q$
- To determine the equation for the axis of symmetry, substitute the axis of symmetry for *x* and *y*.

Practice Exercise

Sketch the following on a set of axis and answer the questions that follows.

1.1 $f(x) = \frac{3}{x} - 2$

a) Write down the range of f
b) Write down the horizontal asymptote of f
c) Determine the value of x for which $f(x) < 0$
d) Write down the equation of h, where $h(x) = f(x) + 2$

1.2 $(x) = -\frac{1}{x} - 1$

a) Write down the range of f
b) Write down the horizontal asymptote of f
c) Determine the value of x for which $g(x) < 0$
d) Write down the equation of h, where $h(x) = -g(x)$

1.3 $f(x) = \frac{4}{x} + 5$

a) Write down the range of f
b) Write down the horizontal asymptote of f
c) Determine the value of x for which $g(x) < 0$
d) Write down the range of h, where $h(x) = f(x) - 6$

1.4 $g(x) = \frac{2}{x} + 3$

a) Write down the range of g
b) Write down the horizontal asymptote of g
c) Determine the value of x for which $g\ (x)\ > 0$
d) Write down the equation of h, where $h\ (x) = g(x) + 3$

3 Exponential Function

An exponential graph is a graph where the x (input value) is the exponent of the equation. When the exponential graph is in its most basic form it will always cross the y-axis at the point (0; 1), and the graph has an asymptote of y = 0.

The Exponential graph equation: $y = a.b^x + q$

The effect of a

- Determine the quadrant
- Indicates the position of the curve relative to the asymptote

The effect of b

- Indicates the direction of the graph

The effect of q

- indicates the horizontal asymptote $y = q$
- indicates when the function will move up or down
- assists in finding the range

Asymptotes (Electric fence)

- There is only one asymptote for the exponential graph.
- The horizontal asymptote is y = q.

Domain and Range

- The domain includes all possible x values, domain can be written as $-\infty < x < \infty$.
- For the range, we need to pay attention to the value of a, and q.
  - ✓ When $a > 0$ (positive), then the range is given as $q < y < \infty$
  - ✓ When $a < 0$ the graph is reflected about the asymptote y = q, and the range becomes $-\infty < y < q$
  - ✓ Axis of Symmetry – none

Practice Exercise

Sketch the following on a set of axes and answer the questions that follow.

1.1 $f(x) = 2^x$

a) Write down the range of f

b) Write down the asymptote of f

c) Determine the value of x for which $g\ (x)\ > 0$

d) Write down the equation of h, where $h\ (x) = g(x) + 1$

1.2 $(x) = \left(\frac{1}{2}\right)$

a) Write down the range of f

b) Write down the asymptote of g

c) Determine the value of x for which $g(x) > 0$

d) Write down the equation of h, where $h(x) = \ -g\ (x) + 1$

1.3 $f(x) = \ 3^x + 2$

a) Write down the range of f

b) Write down the asymptote of f

c) Determine the value of x for which $f\ (x)\ > 0$

d) Write down the range of $h,$ where $h\ (x) = -\ f(x)$

1.4 $g(x) = \ 4^x - 1$

a) Write down the range of g

b) Write down the asymptote of g

c) Determine the value of x for which $g\ (x)\ > 0$

d) Write down the equation of h, where $h\ (x) = g(x) - 2$

MARATHON QUESTIONS

Question 1(Nov 2015)

$f(x) = -2x^2 + 2$ *and* $g(x) = 2^x + 1$ *are the defininig equations of grphs* f *and* g

1.1 Write down an equation for the asymptote of g. (1)

1.2 Sketch the graphs of f and g on the same set of axes, clearly showing ALL intercepts with the axes, turning points and asymptotes. (6)

1.3 Write down the range of f. (1)

1.4 Determine the maximum value of h *if* $h(x) = 3^{f(x)}$. (2)

1.5 What transformation does the graph of y = f (x) undergo in order to obtain the graph of

$y = 2x^2 - 2$ (2)

Question 2 (Nov 2019)

The graphs defined as $g(x) = ax^2 + q$ *and* $f(x) = k^x - 4$ both pass through (-2;0) on the $x - axis$. The $y - intercept\ of\ g$ is (0;-5)

2.1 Write down the:

2.1.1 Equation of the asymptotes of f (1)

2.1.2 Value of q (1)

2.1.3 Coordinates of the other $x - intercepts\ of\ g$ (1)

2.2 Determine the equation of:

2.2.1 g (2)

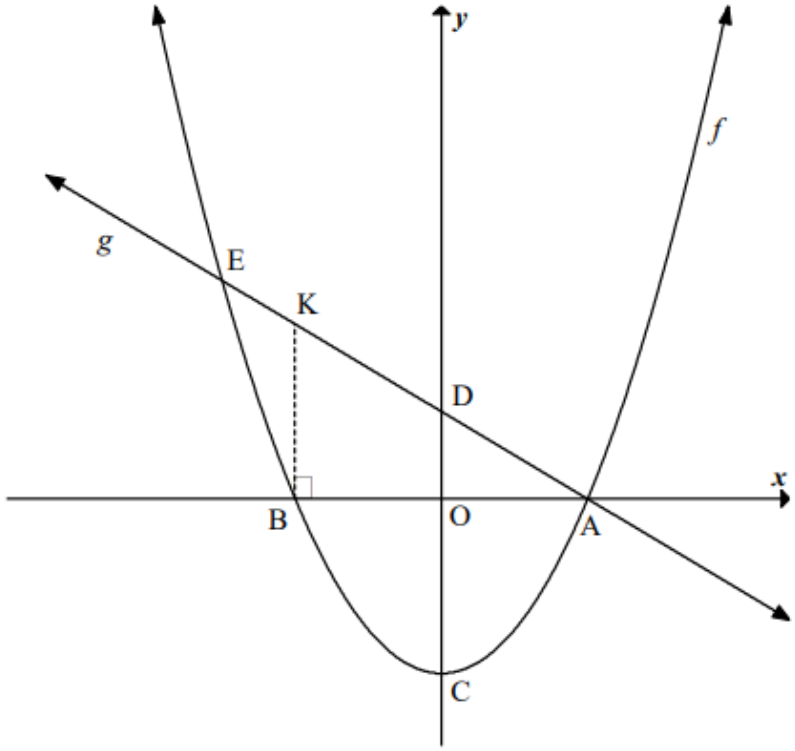
2.2.2 f (3)

2.3 Calculate the $y - intercept\ of\ f$. (2)

2.4 On the same system of axes, sketch the graphs of f and g. Show ALL the intercepts with the axes and asymptotes. (6)

Question 3 (Nov 2016)

The graph of $f(x) = x^2 - 4$ *and* $g(x) = -x + 2$ are sketched below. A and B are the $x - intercepts$ of f. C and D are the $y - intercepts$ of f and g respectively. K is a point on g such that BK $\parallel x - axis. f$ *and* g intersect at A and E.



3.1 Write down the coordinates of C. (1)

3.2 Write down the coordinates of D. (1)

3.3 Determine the length of CD. (1)

3.4 Calculate the coordinates of B. (3)

3.5 Determine the coordinates of E, a point of intersection of f and g. (4)

3.6 For which values of x will:

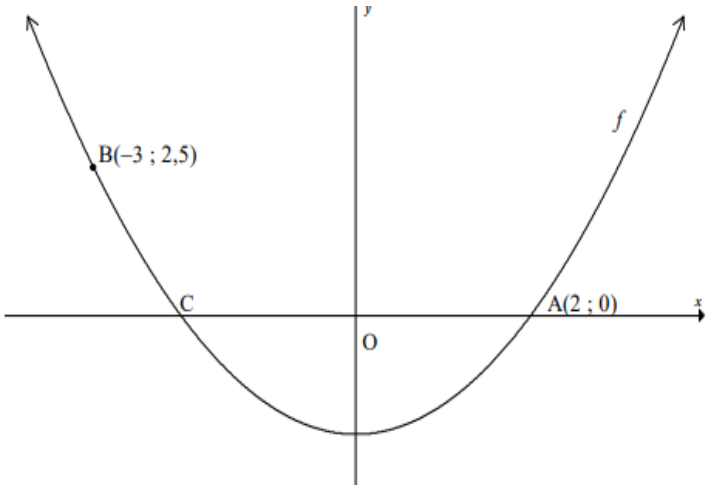
3.6.1 $f(x) < g(x)$ (2)

3.6.2 $f(x).g(x) \geq 0$ (2)

3.7 Calculate the length of AK. (4)

Question 4 (Exemplar 2012)

The graph of $f(x) = ax^2 + q$ is sketched below. Points A(2;0) and B(-3;2,5) lie on the graph of f. Points A and C are $x - intercepts\ of\ f$.



4.1 Write down the coordinates of C. (1)

4.2 Determine the equation of f (3)

4.3 Write down the range of f. (1)

4.4 Write down the range of h, where $h(x) = -f(x) - 2$. (2)

4.5 Determine the equation of an exponential function, $g(x) = b^x + q$, with range $y > -4$ and passes through point A. (3)

Question 5 (Exemplar 2012)

Given: $f(x) = \frac{3}{x} + 1\ and\ g(x) = -2x - 4$

5.1 Sketch the graph of f and g on the same system of axes. (6)

5.2 Write down the equation of the asymptotes of f. (2)

5.3 Write down the domain of f. (2)

5.4 Solve for x if $f(x) = g(x)$ (5)

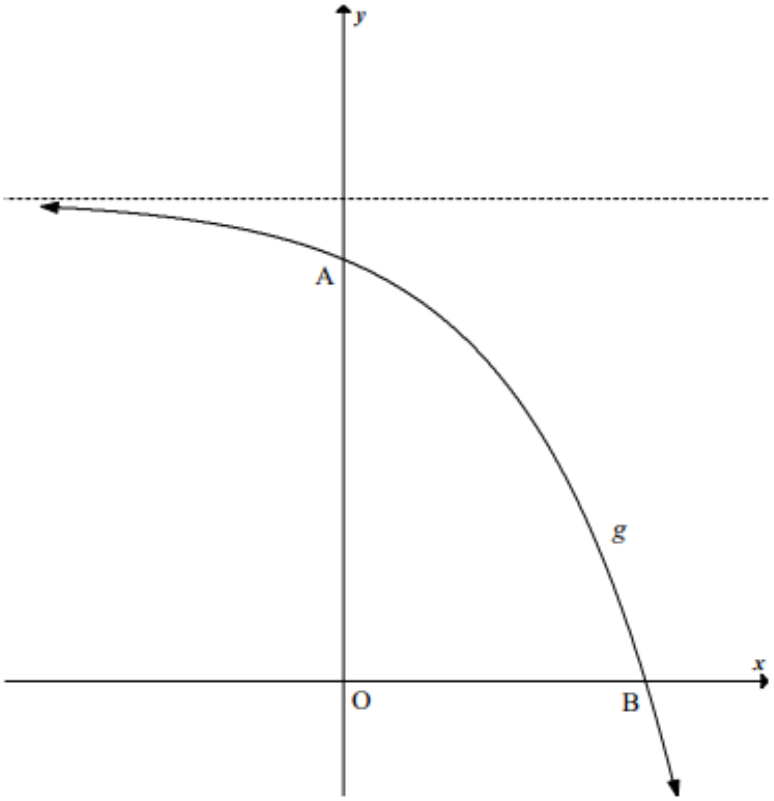
5.5 Determine the values of x if $-1 \leq g(x) < 3$. (3)

5.6 Determine the $y - intercepts$ of k if $k(x) = 2g(x)$ (2)

5.7 Write down the coordinates of the $x - and\ y - intercepts$ of h if h is the graph of g reflected about the y-axis.

Question 6 (Nov 2016)

The graph of $g(x) = -2^x + 8$ is sketched below. A and B are the $y - and\ x - intercepts$ respectively of g.



6.1 Write down the range of g. (1)

6.2 Determine the coordinates of B. (3)

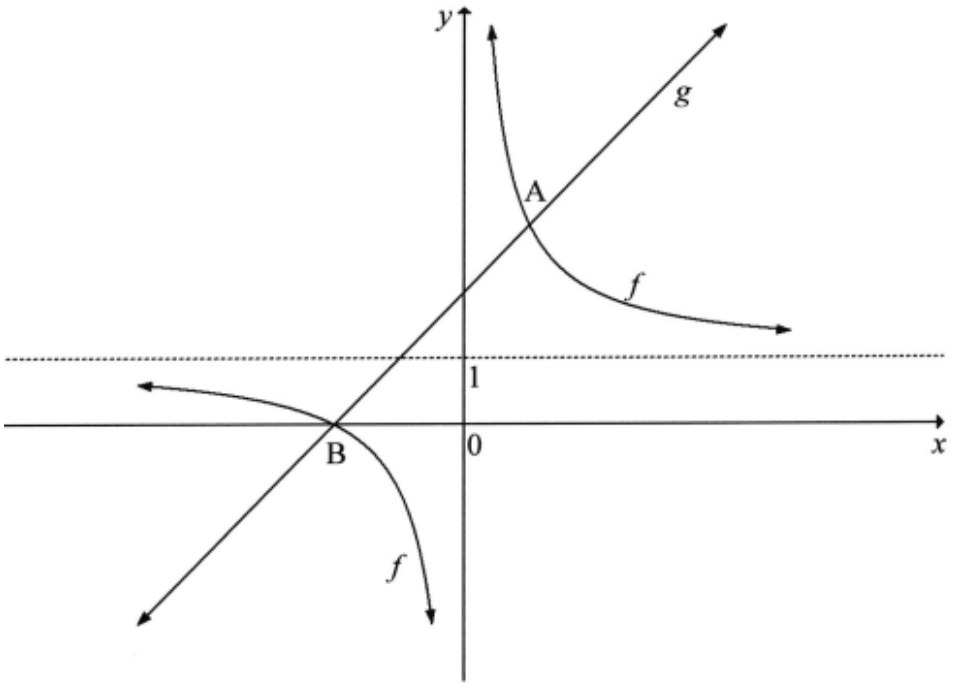
6.3 If g is reflected over the $x - axis$ to form a new graph of h, determine the equation of h. (2)

6.4 Explain why the $x - intercepts\ of\ g\ and\ h$ are both at B. (2)

Question 7

Sketched below are the graphs of $f(x) = \frac{k}{x} + 2$ and $g(x) = x + 2$

- The equation of the horizontal asymptote of f is $y = 1$.
- Graph g cuts the $x - axis$ at B.
- Graph f and g intersect at A and B.



7.1 Write down the:

7.1.1 Value of q. (1)

7.1.2 Domain of f. (2)

7.2 Determine the:

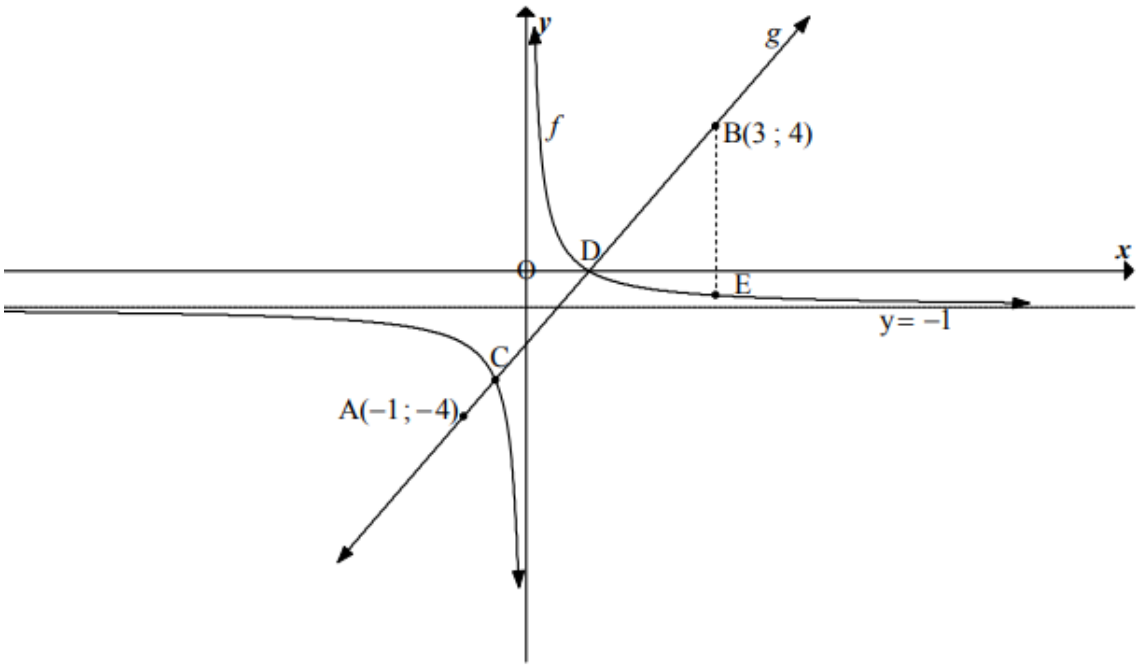
7.2.1 Equation of the line of symmetry of f that has a negative gradient. (2)

7.2.2 Equation of f. (4)

7.2.3 Coordinates of A, a point of intersection of f and g. (5)

Question 8

The sketch below shows f and g, the graphs of $f(x) = \frac{1}{x} - 1$ *and* $g(x) = ax + q$



8.1 Show that $a = 2$ *and* $q = -2$. (2)

8.2 Determine the values of x for which $f(x) = g(x)$. (4)

8.3 For which values of x is $g(x) \geq f(x)$? (3)

8.4 Calculate the length of BE. (3)

8.5 Write down an equation of h if $h(x) = f(x) + 3$ (1)

RAISING THE BAR

Question 1

A hyperbola, h, is described with the following characteristics:

- The equation of the vertical asymptote is $x = 0$.
- The range of h is $(-\infty; 3) \cup (3; \infty)$.
- The $x - intercept$ of h is (2;0)

Determine the equation of h. (4)

Question 2

The function $p(x) = k^x + q$ is described by the following properties:

- $k > 0; k \neq 1$
- $x - intercepts$ at (2;0)
- The horizontal asymptote is $y = -9$

2.1 Write down the range of p. (1)

2.2 Determine the equation of p. (3)

2.3 Sketch the graph of p. Show clearly the intercepts with the axes and the asymptotes. (3)

Question 3

Given: $f(x) = ax^2 + c$,

f passes through the $x - axis$ at $(d - 5) and\ (d - 1)$, where $d \in R$.

3.1 Determine the value of d. (2)

3.2 Determine the value of a and c if it is also given that $f(1) = -9$. (4)

Question 4

Determine the equation of the function $g(x) = \frac{a}{x} + q$, with the asymptote $y = -2$. The straight line $f(x) = -x$ intersects the graph of $g(x)$ at the point (5;-5) (3)

Question 5

The function $h(x) = k^x + q$, is described by the following properties:

- $k > 0; k \neq 1$
- $h(0) = -7$
- $h(3) = 0$
- horizontal asymptote: $y = -8$

Using the information provided, draw a neat sketch graph of $h(x) = k^x + q$ (4)