GRADE 10 TRIGONOMTERY

Summary of the trigonometric ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

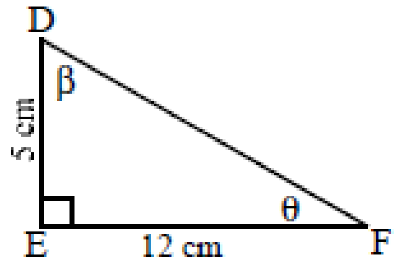
EXAMPLE 1

In ΔDEF, $DE = 5$, $EF = 12$, $\hat{E} = 90°$, $\hat{D} = \beta$ and $\hat{F} = \theta$.

(a) Determine the length of the hypotenuse DF.

(b) Write the value of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

(c) Write the value of $\sin\beta$, $\cos\beta$ and $\tan\beta$.

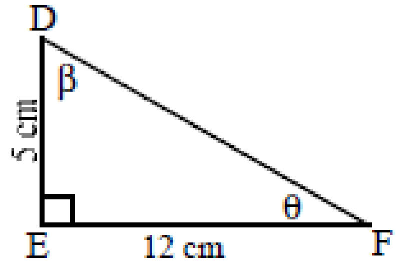


Solutions

(a) $DF^2 = 5^2 + 12^2$ [Pythagoras]

$\therefore DF^2 = 169$

$\therefore DF = 13$ cm



(b) $\sin\theta = \frac{opp}{hyp} = \frac{5}{13}$

$\cos\theta = \frac{adj}{hyp} = \frac{12}{13}$

$\tan\theta = \frac{opp}{adj} = \frac{5}{12}$

(c) $\sin\beta = \frac{opp}{hyp} = \frac{12}{13}$

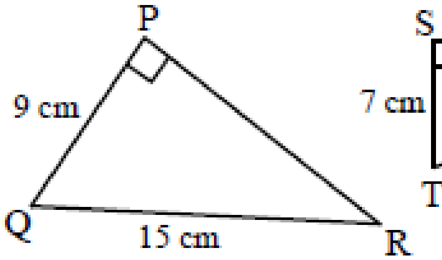
$\cos\beta = \frac{adj}{hyp} = \frac{5}{13}$

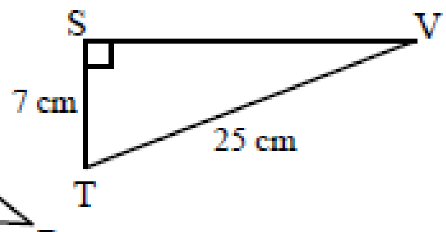
$\tan\beta = \frac{opp}{adj} = \frac{12}{5}$

EXAMPLE 2

Determine: (a) $\tan Q$

(b) $\cos V$





Solutions

(a) $PR^2 = 15^2 - 9^2$

$\therefore PR^2 = 144$

$\therefore PR = \sqrt{144} = 12\,\text{cm}$

$\therefore \tan Q = \frac{opp}{adj} = \frac{12}{9} = \frac{4}{3}$

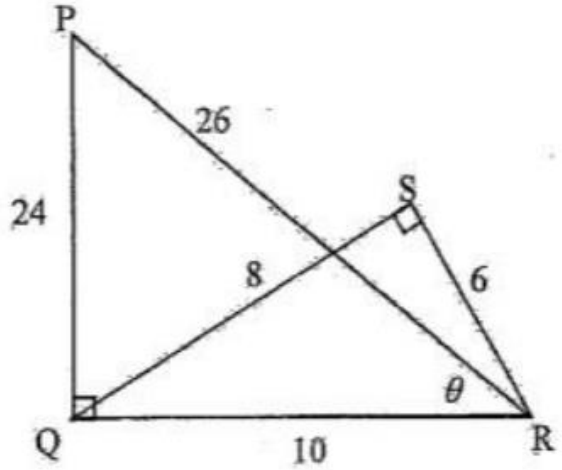
(b) $SV^2 = 25^2 - 7^2$

$\therefore SV^2 = 576$

$\therefore SV = \sqrt{576} = 24\,\text{cm}$

$\therefore \cos V = \frac{adj}{hyp} = \frac{24}{25}$

ΔPQR and ΔSQR are right-angled triangles as shown in the diagram below.
$PR = 26$, $PQ = 24$, $QS = 8$, $SR = 6$, $QR = 10$ and $P\hat{R}Q = \theta$.



4.1 Refer to the diagram above and, WITHOUT using a calculator, write down the value of:

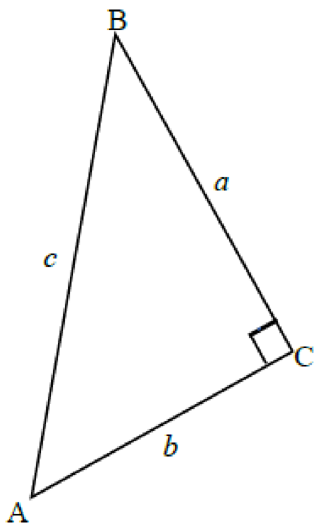
4.1.1 $\tan \hat{P}$ (1)

4.1.2 $\sin S\hat{Q}R$ (1)

4.1.3 $\cos\theta$ (1)

QUESTION 4

4.1 Triangle ABC with sides *a, b* and *c* and angle $\hat{C} = 90^0$ as shown below:



4.1.1 Write the following in terms of *a*, *b* and *c*:

(a) $\cos A$ (1)

(b) $\tan B$ (1)

(c) $\text{cosec } A$ (2)

4.1.2 If it is given that $a = 5$ and $\widehat{A} = 50^0$, determine the numerical value of *b*. (2)

4.2 Given that $\widehat{A} = 38{,}2°$ and $\widehat{B} = 146{,}4°$ determine the value of $2\cot A + \cos 3B$. (3)

4.3 If it is given that $13\cos\beta = 12$ and β is an acute angle, determine with the aid of a diagram the value of the following:

4.3.1 $\sin\beta$ (3)

4.3.2 $\tan\beta$ (1)

TRIGONO ETRIC EQUATIONS

SOLVING SIMPLE TRIGONOMETRIC EQUATIONS

From the previous example and exercise, we saw that the trigonometric ratio of a given angle can easily be found by using a calculator. The reverse process is also possible. This process is **finding the size of an angle** when given its sine, cosine or tangent ratio.

Consider the trigonometric equation $\cos\theta = 0,5$. We want to find the size of the angle that will result in the ratio 0,5.

In order to do this, we will make use of the $\cos^{-1}$ function on the calculator. This function can be found above the "cos" button. In order to use this function, you have to press the "SHIFT" button on your calculator.

Therefore, the sequence of buttons to press on most of the calculators will be as follows:

- Press the SHIFT button followed by the "cos" button. The $\cos^{-1}$ will appear on your calculator screen.
- Enter the ratio value (in this case 0.5)
- Press the "=" button (some calculators expect you to close the brackets first)

Some calculators may have the button INV or 2nd F instead of SHIFT.

Always ensure that your calculator is on the degree mode [DEG]

EXAMPLE 5

(a) Determine the size of the acute angle θ in each of the following trigonometric equations. Round your answers off to one decimal place where necessary.

(1) $\cos\theta = 0,5$ (2) $\tan\theta = 4,123$ (3) $\sin\theta = 0,707$

(b) Solve the following equations. Round your answers off to one decimal place where necessary. All angles are acute.

(1) $2\sin\theta = 1,124$ (2) $\sin 2\theta = 0,435$

(3) $\frac{1}{2}\tan 2x = 3$ (4) $1 + 2\cos(x + 10°) = 2,356$

Solutions

(a) (1) $\cos\theta = 0,5$ press [shift] [cos] then 0,5

$\therefore \theta = \cos^{-1}(0,5)$ as it appears on the calculator screen

$\therefore \theta = 60°$

(2) $\tan\theta = 4,123$ press [shift] [tan] then 4,123

$\therefore \theta = \tan^{-1}(4,123)$ as it appears on the calculator screen

$\therefore \theta = 76,4°$

(3) $\sin\theta = 0,706$ press [shift] [sin] then 0,706

$\therefore \theta = \sin^{-1}(0,706)$ as it appears on the calculator screen

$\therefore \theta = 44,9°$

(b) (1) $2\sin\theta = 1,124$ $\sin\theta$ has been multiplied by 2

$\therefore \sin\theta = 0,562$ isolate $\sin\theta$ by dividing by 2

$\therefore \theta = \sin^{-1}(0,562)$

$\therefore \theta = 34,2°$

(2) $\sin(2\theta) = 0,435$ insert the brackets

$\therefore 2\theta = \sin^{-1}(0,435)$

$\therefore 2\theta = 25,78529...°$ determine θ

$\therefore \theta = 12,9°$ divide by 2 and then determine θ

(3) $\frac{1}{2}\tan 2x = 3$ LCD: 2

$\therefore 2 \times \frac{1}{2}\tan 2x = 3 \times 2$ isolate $\tan 2x$

$\therefore \tan 2x = 6$

$\therefore 2x = \tan^{-1}(6)$

$\therefore 2x = 80,537...°$

$\therefore x = 40,3°$

(4) $1+2\cos(x+10^\circ)=2,356$

$\therefore 2\cos(x+10^\circ)=1,356$

$\therefore \cos(x+10^\circ)=0,678$

$\therefore x+10^\circ=\cos^{-1}(0,678)$

$\therefore x+10^\circ=47,3124...^\circ$

$\therefore x=37,3^\circ$

EXERCISE 3

(a) Determine the size of the acute angle θ in each of the following, rounding your answers off to two decimal places where necessary.

(1) $\sin\theta=0,866$ (2) $\cos\theta=0,866$ (3) $\tan\theta=1,703$

(4) $\sin\theta=1$ (5) $\cos\theta=1$ (6) $\tan\theta=1$

(b) Solve the following equations by finding the size of the acute angle θ in each case rounding your answers off correct to one decimal place where necessary.

(1) $\cos 3\theta=0,33$ (2) $3\cos\theta=0,33$ (3) $\sin 4\theta=0,888$

(4) $4\sin\theta=0,888$ (5) $\tan 4\theta=4$ (6) $4\tan 4\theta=4$

(c) Determine the value of the acute angle x in each of the following equations.

(1) $\sin(x-20^\circ)=0,678$ (2) $3\cos(x+30^\circ)=2,121$

(3) $2\tan(2x-10^\circ)=3,4641$ (4) $2\tan 2x-10=3,4641$

4.4 Solve for θ (correct to TWO decimal places) if θ is an acute angle:

4.4.1 $\tan\theta = 2,9$ (1)

4.4.2 $2\cos\theta + 5 = 6$ (3)

ANGLES IN THE CARTESIAN PLANE

In this section, we will extend the trigonometric definitions to include angles in the interval $[0°\,;360°]$.

Consider a circle with centre $O(0\,;0)$ and radius r with $R(x\,;y)$ any point on the circle. θ is the angle measured anti-clockwise from the positive side of the x-axis to the radius OR, which is referred to as the **terminal arm** and θ is said to be in **standard position**.

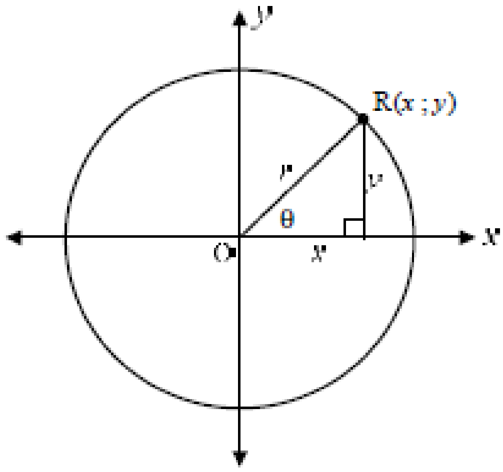
Note that for every point $R(x\,;y)$ on the circumference of the circle, $x^2+y^2=r^2$

For each point $R(x\,;y)$ on the terminal arm of θ, the following trigonometric functions of θ are defined:

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{y}{r}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{x}{r}$$

$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{y}{x}$$



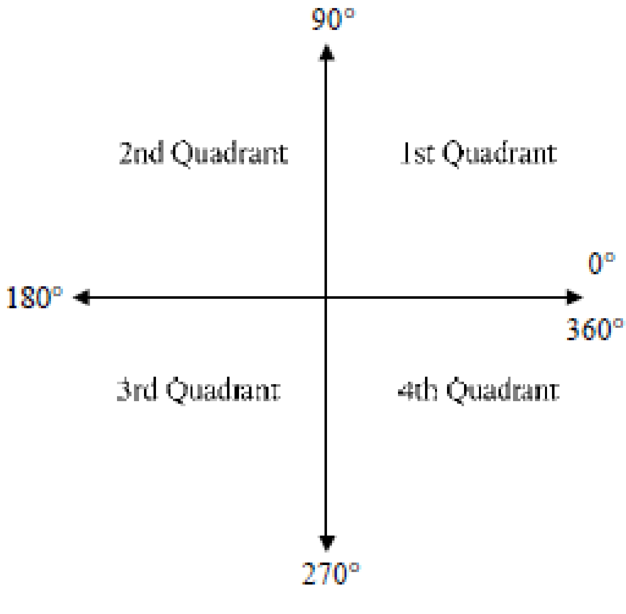
The Cartesian plane is divided into four quadrants allowing for angles in the interval $(0°\,;360°)$

Angles in the first quadrant will lie in the interval $(0°\,;90°)$

Angles in the second quadrant will lie in the interval $(90°\,;180°)$

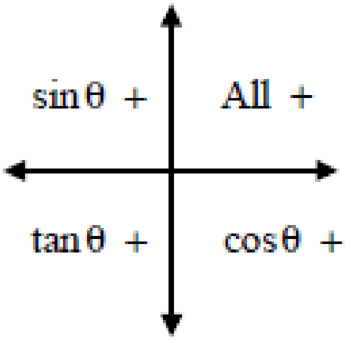
Angles in the third quadrant will lie in the interval $(180°\,;270°)$

Angles in the fourth quadrant will lie in the interval $(270°\,;360°)$



Conclusion

- All trigonometric functions are positive in the first quadrant.
- $\sin\theta$ is positive in the second quadrant and $\tan\theta$ and $\cos\theta$ are negative.
- $\tan\theta$ is positive in the third quadrant and $\sin\theta$ and $\cos\theta$ are negative.
- $\cos\theta$ is positive in the fourth quadrant and $\sin\theta$ and $\tan\theta$ are negative.



Here is a useful way of remembering this rule of signs in the quadrants:

Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4
All	singers (sin)	take (tan)	cough sweets (cos)

EXERCISE 7

In which quadrant does the terminal arm of the angle θ lie if:

(a) $\sin\theta > 0$ and $\cos\theta > 0$ (b) $\sin\theta < 0$ and $\cos\theta < 0$

(c) $\tan\theta > 0$ and $\cos\theta < 0$ (d) $\tan\theta < 0$ and $\cos\theta < 0$

(e) $\sin\theta < 0$ and $\theta \in [90°;270°]$ (f) $\cos\theta < 0$ and $0° < \theta < 180°$

EXAMPLE 13

If $\cos\theta = \frac{4}{5}$ and $\theta \in [0°;90°]$, calculate without the use of a calculator and with the aid of a diagram the value of $\tan^2\theta$.

Solution

Given: $\cos\theta = \frac{4}{5} = \frac{x}{r}$

and $x^2 + y^2 = r^2$.......Pythagoras

$\therefore (4)^2 + y^2 = (5)^2$

$\therefore 16 + y^2 = 25$

$\therefore y^2 - 9 = 0$

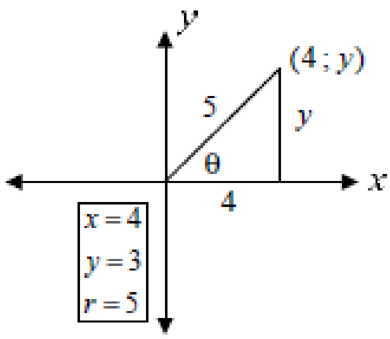
$\therefore (y+3)(y-3) = 0$

$\therefore y = \pm 3$

But y is positive in Quadrant 1

$\therefore y = 3$

$\therefore \tan^2\theta = \left(\frac{y}{x}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$



Quicker method to solve for y:

$y^2 - 9 = 0$

$\therefore y^2 = 9$

$\therefore y = \pm\sqrt{9}$

$\therefore y = \pm 3$

EXAMPLE 14

If $13\sin\theta = -5$ and $\theta \in [90°;270°]$ calculate without the use of a calculator and with the aid of a diagram the value of $\cos\theta + \sin\theta$.

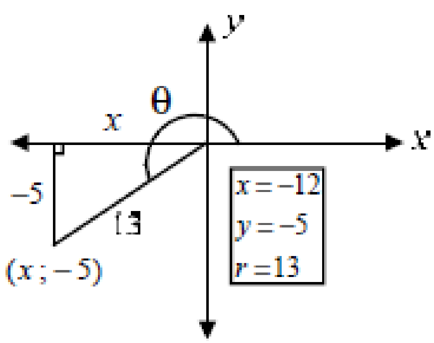
Solution

$13\sin\theta = -5$

$\therefore \sin\theta = -\frac{5}{13} = \frac{-5}{13} = \frac{y}{r}$ (r is always positive)

$\sin\theta$ is negative and therefore the terminal arm will lie in the **third or fourth** quadrant.

But with $\theta \in [90°;270°]$, the terminal arm will lie in the **third** quadrant.



$x^2 + y^2 = r^2$.......Pythagoras

$\therefore x^2 + (-5)^2 = (13)^2$

$\therefore x^2 + 25 = 169$

$\therefore x^2 - 144 = 0$

$\therefore (x+12)(x-12) = 0$

$\therefore x = \pm 12$

But x is negative in Quadrant 3

$\therefore x = -12$

Quicker method:

$x^2 - 144 = 0$

$\therefore x^2 = 144$

$\therefore x = \pm\sqrt{144}$

$\therefore x = \pm 12$

$\cos\theta + \sin\theta$

$= \left(\frac{-12}{13}\right) + \left(\frac{-5}{13}\right)$

$= \frac{-17}{13} = -1\frac{4}{13}$

EXERCISE 8

(a) If $\sin\theta = \frac{3}{5}$ and $0° \le \theta \le 90°$, determine by means of a diagram:

(1) $\cos^2\theta$ (2) $2\tan\theta$

(b) If $\tan\theta = \frac{5}{12}$ and $\sin\theta > 0$, determine by means of a diagram:

(1) $13\cos\theta$ (2) $\cos^2\theta + \sin^2\theta$

(c) If $5\cos A + 3 = 0$ and $180° < A < 360°$, determine by means of a diagram:

(1) $\tan^2 A$ (2) $\frac{\sin A}{\cos A}$

(d) If $8\tan\theta + 15 = 0$ and $\theta \in [90°; 270°]$, determine by means of a diagram:

(1) $\sin\theta + \cos\theta$ (2) $34\sin\theta - 17\cos\theta$

(e) If $13\cos\theta - 5 = 0$ and $180° \le \theta \le 360°$, determine by means of a diagram:

(1) $\sin^2\theta + \cos^2\theta$ (2) $25\tan^2\theta$

(f) If $4\tan B - 3 = 0$ and $\cos B < 0$, determine by means of a diagram:

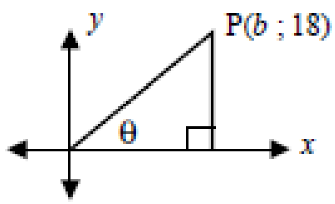
(1) $(\sin B + \cos B)^2$ (2) $25(\sin B - \cos B)^2$

(g) If $2\sin\theta + 1 = 0$ and $90° < \theta < 270°$ calculate without the use of a calculator and with the aid of a diagram the value of the following:

(1) $4\cos^2\theta$ (2) $81\tan^2\theta$

(h) In the diagram alongside $\tan\theta = \frac{12}{5}$ and $P(b\,;\,18)$

Determine the value of b without using a calculator.



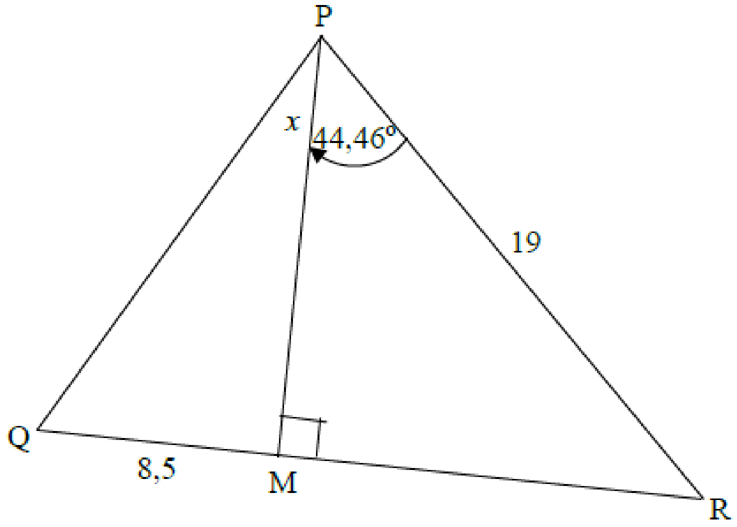
(i) If $\tan\theta = \frac{a}{b}$ where $\theta \in [0°; 90°]$, determine $\sin^2\theta$ by means of a diagram.

PAST EXAMINATION QUESTIONS: MIXED CONCEPTS

QUESTION 6

6.1 Drawn below is triangle PQR with PM ⊥ QR. $M\hat{P}R = 44{,}46°$ and $Q\hat{P}M = x$.

PR = 19 units and QM = 8,5 units.



Determine the value of:

6.1.1 x (4)

6.1.2 PQ (2)

QUESTION 4

4.1 Given $4 \cot \theta + 3 = 0$ and $0° < \theta < 180°$

4.1.1 Use a sketch to determine the value of the following.

DO NOT use a calculator.

(a) $\cos\theta$ (4)

(b) $\dfrac{3\sin\theta\sec\theta}{\tan\theta}$ (4)

4.1.2 Hence, show that $\sin^2\theta - 1 = -\cos^2\theta$ (3)

4.2 Simplify the following expression without the use of a calculator:

$\cos 30° \tan 60° + \text{cosec}^2 45° \sin^2 60°$

(3)

4.3 Solve for θ correct to TWO decimal places where $0° < \theta < 90°$

4.3.1 $\dfrac{4}{3}\sin\theta = \cos 37°$ (3)

4.3.2 $\dfrac{\text{cosec }\theta}{3} = 2$ (3)

[20]

QUESTION 4

4.1 If $x = 75°$ and $y = 126{,}5°$, evaluate the following correct to TWO decimal places.

4.1.1 $\sin(y - x)$ $\sin(y-x)$

(2)

4.1.2 $\cot y$ (2)

4.2 Determine the value of β correct to ONE decimal place.

4.2.1 $\tan\beta + 0{,}25 = 1$ (2)

4.2.2 $5\sec\frac{\beta}{3} = 7$ (3)

4.2.3 $8\cos(2\beta + 30°) = 4$ (4)

[13]

QUESTION 5

5.1 Given that $-3\cos\theta - 2 = 0$ and $\sin\theta > 0$. Determine WITHOUT the use of a calculator and with the aid of a diagram:

5.1.1 $\sin\theta$ (4)

5.1.2 $4\tan^2\theta + 2$ (2)

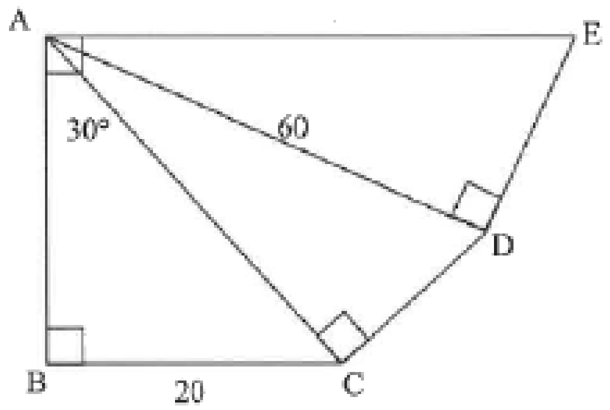
5.2 Simplify the following expression WITHOUT using a calculator.

$$\frac{\tan 45°.\sin 60°}{\tan 30°\left(1-\sin 30°\right)}$$ (5)

[11]

QUESTION 4

4.1 In the diagram below, ABC, ACD and ADE are right-angled tirangles. $B\hat{A}E = 90°$ and $B\hat{A}C = 30°$. BC = 20 units and AD = 60 units.



Calculate the:

4.1.1 Length of AC (2)

4.1.2 Size of $C\hat{A}D$ (2)

4.1.3 Length of DE (3)

4.2 Solve for x, correct to ONE decimal place, where $0° \leq x \leq 90°$:

4.2.1 $\tan x - 2{,}01$ (2)

4.2.2 $5\cos x + 2 = 4$ (3)

4.2.3 $\dfrac{\operatorname{cosec} x}{2} = 3$ (3)

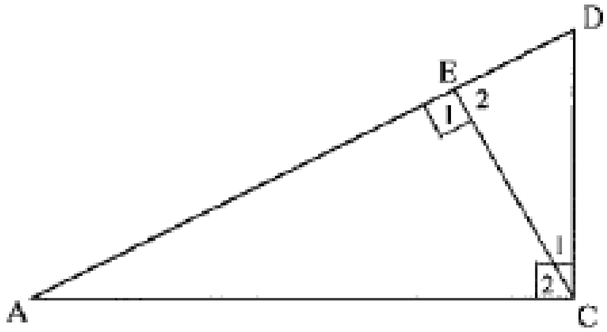
[15]

QUESTION 3

3.1 If $x = 37°$ and $y = 44°$, calculate the value of $\sin^2 x + 2\cos y$. (1)

3.2 WITHOUT using a calculator, determine the value of $\frac{\sin 30°.\cot 45°}{\cos 30°.\tan 60°}$ (3)

3.3 In the diagram below, ΔACD is right-angled at C. E lies on AD such that CE is perpendicular to AD.



3.3.1 Write down the ratio for $\cos D$ in ΔACD. (1)

3.3.2 Write down the ratio for $\cos D$ in ΔCED. (1)

3.3.3 If AD = 13 units and DC = 5 units, calculate the length of ED. (2)

3.4 Given that $\cos\theta = \frac{5}{13}$ and $\sin\theta < 0$.

With the aid of a diagram and WITHOUT using a calculator, determine the value of:

3.4.1 $\sin\theta$ (3)

3.4.2 $\sec\theta + \tan^2\theta + 1$ (4)

[15]

QUESTION 4

4.1 If $0° \le \theta \le 90°$, solve for θ in each of the following questions:

4.1.1 $2\sin\theta + 1 = 1{,}28$ (2)

4.1.2 $\tan 2\theta = 4\cot 60^0$ (3)