Chapter 1 VECTORS IN TWO DIMENSIONS

RESULTANT OF PERPENDICULAR VECTORS

REVISION OF IMPORTANT CONCEPTS FROM GRADE 10

In Grade 10 you were acquainted with scalars and vectors. Let's do a quick revision of the most important concepts regarding scalars and vectors.

Scalar quantity

Scalars are physical quantities, consisting of magnitude only.

Certain scalars are numbers only (numerical magnitudes), such as 1, 2, 3, etc. without units. In natural sciences we specify scalars more completely with a number and the applicable unit such as a mass of 20 kg, a velocity of 5 m·s^{-1} or a charge of 20 C.

We can add and subtract scalar magnitudes, just as we can do with ordinary numbers. For example, a distance of 14 m + 13 m = 27 m and a velocity of 20 m·s^{-1} - 4 m·s^{-1} = 16 m·s^{-1}.

Vector quantity

Vectors are physical quantities, possessing magnitude and direction.

A vector quantity is indicated by a number (numerical magnitude), a unit and a direction. For instance, a velocity of 4 m·s^{-1} to the right, a force of 20 N Easterly and a displacement of 10 m upwards

When adding vectors, we have to take the direction of each vector into account. When two people push a crate in the same direction, the effect is not the same as when they are pushing in opposite directions.

VECTOR QUANTITIES

Physical quantities	Quantity symbol	SI unit of measurement	Symbol for unit
displacement	x	meter	m
velocity	v	meter per second	m·s^{-1}
acceleration	a	meter per second squared	m·s^{-2}
power	F	newton	N
weight	w	newton	N

SCALAR QUANTITIES

Physical quantities	Quantity symbol	SI unit of measurement	Symbol for unit
time	t	seconds	s
mass	m	kilogram	kg
distance	D	meter	m
speed	v	meter per second	m·s^{-1}
charge	Q	coulomb	C

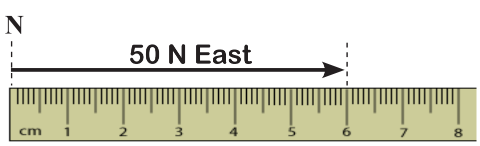
Graphic presentation of vectors

A vector quantity is indicated by a **straight line** with an **arrow point**.

Parts of a vector

- The **length** of the line represents the magnitude of the vector magnitude.
- The **arrow** head represents the **direction** of the vector magnitude.
- The **starting** point of the vector is called the **tail**, and the end, indicated by the arrow head, is called the **head**.
- The magnitude and direction of a vector can be represented more accurately when plotted to scale. Scale: 1 cm = 10 N

The graphic representation of a vector is called a **vector diagram**.



Symbolic representation of scalars and vectors

Letters of the alphabet are used for both scalars and vectors. However, because vectors (which have magnitude and direction) differs from scalars (which only have magnitude), their notations differ.

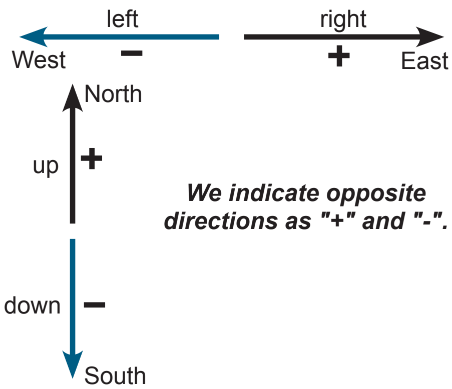
- **Scalars** are indicated by **ordinary letters, e.g. m** for mass and **D** for distance.
- **Vectors** are indicated by **bold letters, e.g. F** for force, **or in ordinary letters with an arrow** directly above it, e.g. $\vec{F}$ for force.
- The magnitude of a vector is written in **ordinary letters** as in the case of scalars. E.g. **F** for the magnitude of a force.

Direction of vectors

In Grade 10 we represented vectors in one dimension (in a straight line). The utilization of positive and negative vectors is useful when working with vectors in one dimension.

Where one direction is indicated as positive, the opposite direction has to be indicated as negative.

- Horizontal direction (left and right, or East and West): Where right or East is regarded as "positive", left or West must be regarded as "negative".
- Vertical direction (up and down or North and South): Where up or North is regarded as "positive", down or South must be regarded as "negative".



In Grade 11 we study **vectors in two dimensions**. Therefore vectors can be denoted in various ways. There are three methods we can use to denote direction.

(1) Bearings

A line of reference is chosen as **north**, with an angle of 0°. All angles are then measured directly by **clockwise** angular rotation (0° - 360°) from the reference line.

Figure 1 shows how this method can be utilised to indicate the following directions:

A = direction 30° or bearing of 30°
B = direction 120° or bearing of 120°
C = direction 270° or bearing of 270°
D = direction 320° or bearing of 320°

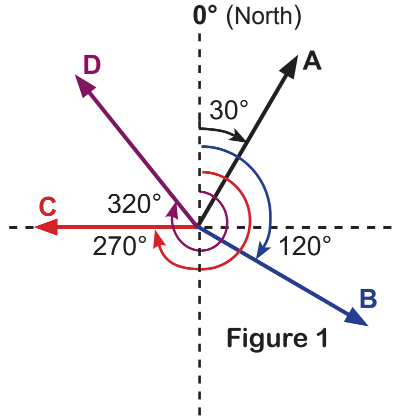


Figure 1

(2) Compass readings

In this method we utilise the directions of wind, **north**, **south**, **east** and **west**.

Figure 1 shows how this method can be implemented to indicate the following directions:

A = 60° east of north, or N 60° O
B = 20° west of south, or S 20° W
C = 30° west of north, or N 30° W

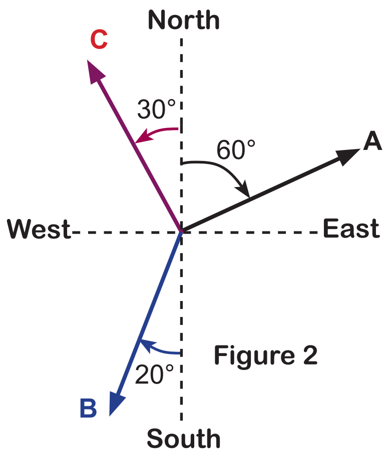


Figure 2

(3) x- and y-axes

When the x- and y-axes are used to indicate direction, the angle is measured in relation to the x-axis. It is also imperative to indicate whether the angle is above or underneath the x-axis, and whether the angle was measured in relation to the positive or negative x-axis.

Figure 3 shows how this method can be implemented to indicate the following directions:

A = 30° above the positive x-axis
B = 45° underneath the positive x-axis
C = 37,5° underneath the negative x-axis

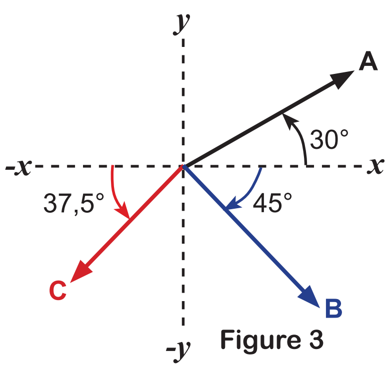


Figure 3

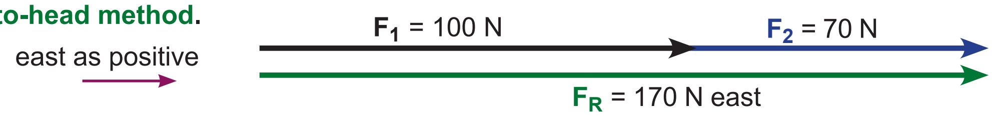
Addition of vectors

Two or more vectors can be added together graphically or algebraically to produce a single vector as answer, which has the same effect as the two or more separate vector magnitudes. This combined effect of two or more vectors is called the **resultant** (of resultant vector **R**).

> ***The resultant of a number of vectors is the single vector which has the same effect as the original vectors combined.***

The resultant of two or more forces $\mathbf{F_1}$ and $\mathbf{F_2}$ can be indicated as a **vector equation**: $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$. Remember: Both the **magnitude and the direction** of each vector have to be taken into account.

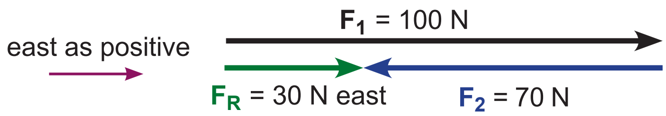
Two forces $\mathbf{F_1}$ of 100 N east and $\mathbf{F_2}$ of 70 N east are algebraically calculated as $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$ = 100 N + 70 N = 170 N east. The following figure indicates how these two force vectors can be added together graphically by sketching the force vector $\mathbf{F_2}$ with its tail connected to the head of force vector $\mathbf{F_1}$. This graphical method of vector addition is called the **tail-to-head method**.



When forces are exerted in opposite directions, they are still added together algebraically. With the equation **F_1** 100 N east but **F_2** altered to 70 N west, the easterly direction is now indicated as positive; the westerly direction is then negative. **F_2** is in the opposite direction as **F_1** and thus **F_2** is then called the **negative vector** so that:

$F_R = F_1 + F_2$ = (+100 N) + (-70 N) = 30 N east.

The following diagram indicates how these two force vectors can be added together graphically by drawing the force vector **F_2** with its tail connected to the head of force vector **F_1**, but in the opposite direction.



We always indicate the resultant vector by the number representing its magnitude, the correct unit and the correct direction.

Exercise 1 REVISION OF IMPORTANT CONCEPTS OF GRADE 10

1. Explain the concepts and provide three examples of each: **scalar quantities** and **vector quantities**

   Scalar: ______________________________

   ____

   Vector: ______________________________

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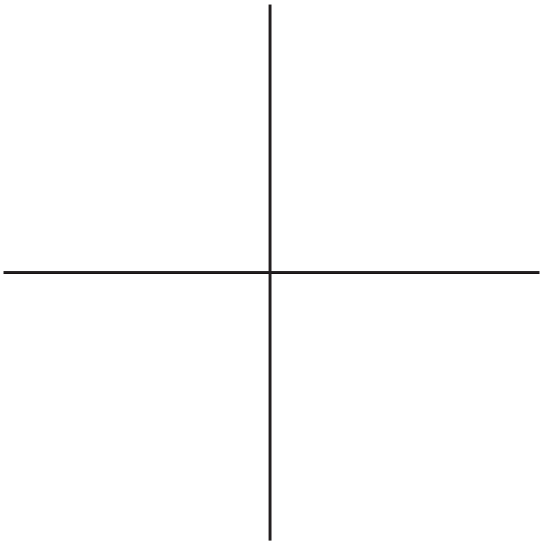
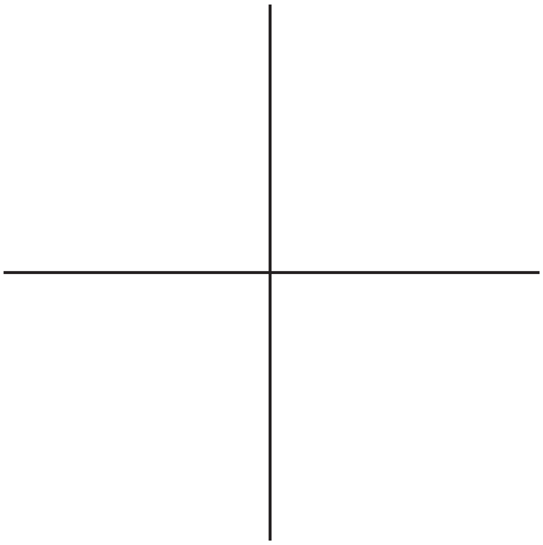
2. Take 0˚ (North) as reference line. Use lines measuring 4 cm with arrow heads to indicate the following directions on the same axes system:

   A = direction 45° ; B = direction 110°
   C = direction 180° ; D = direction 330°



3. Indicate the wind directions on the following axes system. Use lines measuring 4 cm with arrow heads to indicate the following directions on the same axes system:

   A = 60° W of N ; B = South-East
   C = 35° S of W ; D = 55° N of O



4. Describe each of the directions A, B and C in Question 2, as well as A,C and D in Question 3 in terms of an angle measured in relation to the positive or negative *x*-axis.

   For Question 2:

   A = ______________________________

   B = ______________________________

   C = ______________________________

   For Question 3:

   A = ______________________________

   C = ______________________________

   D = ______________________________

5. Consider the sketch:

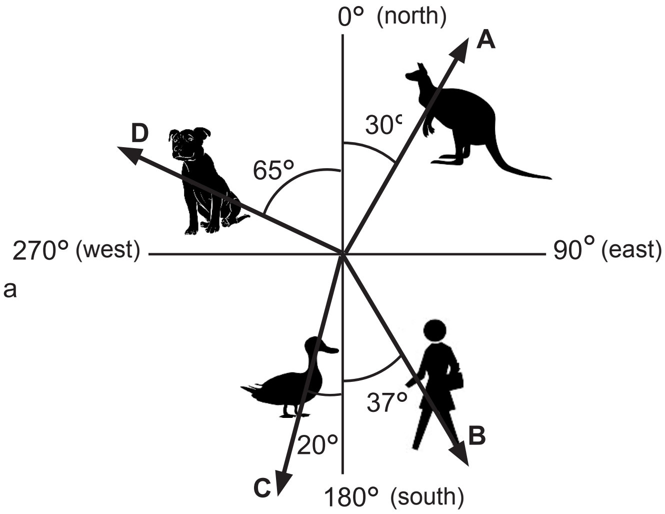
5.1 Indicate the directions of **A**, **B**, **C** and **D** clockwise, measured from the 0˚ (North) reference line.

A = ______________________________

B = ______________________________

C = ______________________________

D = ______________________________



5.2 Indicate the directions of **A**, **B**, **C** and **D** in terms of a sharp angle and the basic wind direction.

A = ______________________________

B = ______________________________

C = ______________________________

D = ______________________________

6. What is meant by the **resultant** of vectors?

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7. A force $\mathbf{F_1}$ of 5 N and a force $\mathbf{F_2}$ of 3 N are simultaneously exerted on an object. The direction of the forces can be altered at will.

7.1 When will these forces have their greatest resultant? ______________________________

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7.2 Confirm your answer in 7.1 by determining the resultant of these forces, utilizing an accurate tail-to-head vector diagram. Use a scale of 10 mm : 1 N.

7.3 Test your answer in Question 7.2 by using a vector equation.

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7.4 When will these forces have their least resultant? ______________________________

____

7.5 Confirm your answers in Question 7.4 by determining the resultant of these forces, utilizing an accurate tail-to-head vector diagram. Use a scale of 10 mm : 1 N

7.6 Test your answers in Question 7.5 by using a vector equation.

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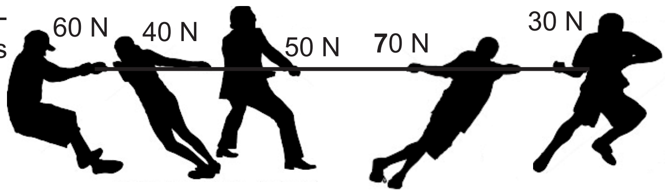
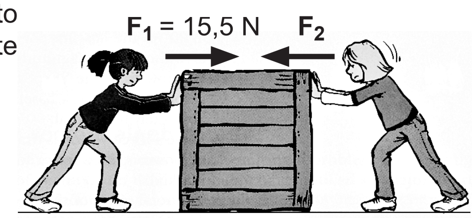
8. Determine the resultant force graphically or by calculation,of all the forces during a tug-of-war competition as illustrated in the sketch. Use a scale of 10 mm:10 N.



9. The following forces are exerted on an object: $\mathbf{F_A}$ = 4,5 N downwards; $\mathbf{F_B}$ = 3,2 N downwards en $\mathbf{F_C}$ = 11,5 N upwards.

   Determine the resultant of the three forces by utilizing a head-to-tail vector diagram. Test your answer by utilizing a vector equation.

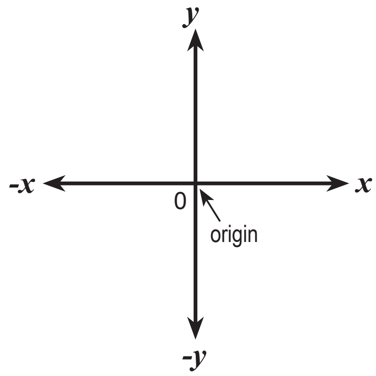
10. Two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$ are exerted on a crate as illustrated. $\mathbf{F_1}$ is 15,5 N to the right. Use a vector diagram as well as a vector equation to calculate $\mathbf{F_2}$ if the resultant of the two forces is 7,3 N is.



THE RESULTANT OF TWO PERPENDICULAR VECTORS

Cartesian co-ordinate system

Vectors can be plotted on a **Cartesian plane**. The Cartesian plane is a set of axes, crossing each other perpendicularly at the origin (0;0). The horizontal axis (x- axis) is positive to the right and negative to the left of the origin; the vertical axis (y-axis) is positive upwards of the origin and negative below the origin.



A Cartesian plane.

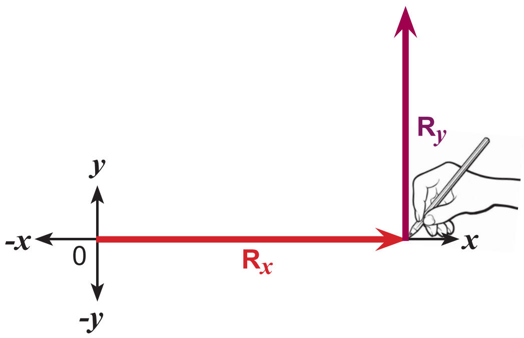
Representation of vectors on the Cartesian plane

- R_x is a vector in the horizontal direction. R_y is a vector in the vertical direction.
- The two vectors are perpendicular (90°) in relation to one another.

Two methods can be utilised to represent these two perpendicular vectors on a Cartesian plane, namely by plotting them **tail-to-head** or by plotting them **tail-to tail**.

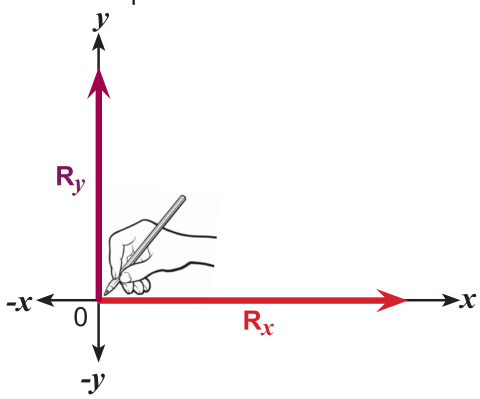
Method 1: Tail-to-head

Starting at the origin, plot vector R_x in the x-direction. Plot vector R_y in the y-direction from the head of R_x, so that the two vectors are plotted **tail-to-head**.



Method 2: Tail-to-tail

Starting at the origin, plot vector R_x in the x-direction. Plot vector R_y in the y-direction from the origin, so that the two vectors are plotted **tail-to-tail**.



Determine the resultants R or R_x and R_y

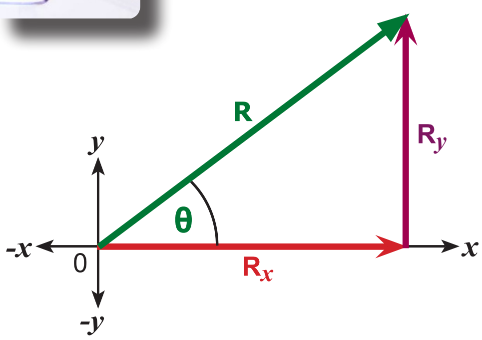
The resultant vector **R** of two or more vectors is the single vector which has the same effect as the original vectors together. **Two methods** can be used to determine the resultant vector **R** of two vectors R_x and R_y which **act upon each other perpendicularly,** namely the **tail-to-head method** and the **tail-to-tail** method.

I. Graphical determination of the resultant vector

1. Tail-to-head method

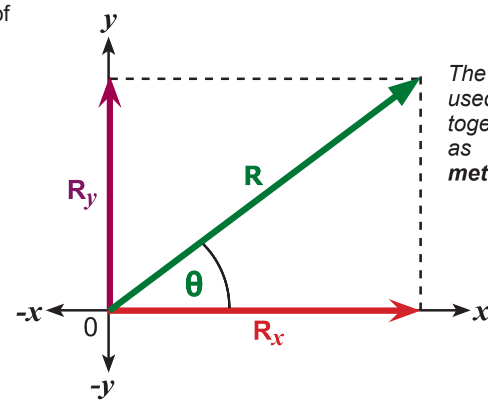
- Obtain a suitable scale, e.g. Scale: 10 mm : 10 N.
- Plot the first vector, R_x according to scale.
- Plot the second vector, R_y according to scale, attaching its tail to the head of R_x
- Connect the tail of R_x with the head of R_y to produce the resultant vector **R**.
- Measure the length of **R** with a ruler and the direction (**θ**) of **R** with a protractor.
- Use the chosen scale to determine the true magnitude of the resultant.
- Describe the direction (**θ**) according to the methods you have learnt to indicate direction.

The tail-to-head method used to add two vectors together, is often referred to as the **triangular method** because the third side of the triangle is formed by the resultant



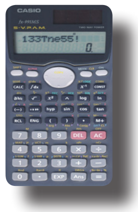
2. Tail-to-tail method

- Obtain a suitable scale, e.g. Scale: 10 mm : 10 N.
- Plot the first vector, R_x according to scale.
- Plot the second vector, R_y according to scale, start at the same origin as R_x (i.e. start at the tail of R_x).
- Complete a rectangle by plotting two lines of equal length parallel to R_x en R_y
- Plot the resultant vector **R** as the diagonal of the rectangle, starting at the tails of R_x and R_y.
- Measure the length of **R** with a ruler and the direction (**θ**) of **R** with a protractor.
- Use the chosen scale to determine the true magnitude of the resultant.
- Describe the direction (**θ**) of **R** according to the methods you have learnt to indicate direction.



*The tail-to-tail method used to add two vectors together, is referred to as the **parallelogram method**, as it forms a parallelogram*

II. Determination of the resultant vector by calculation



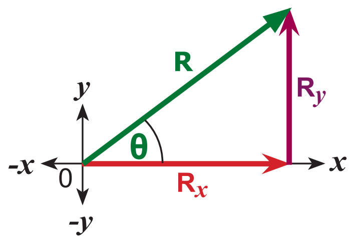
1. Tail-to-head method

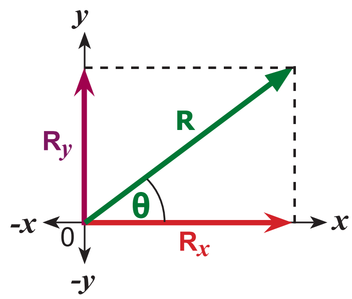
- Plot a sketch-diagram (i.e. a vector diagram which is not according to scale) with vectors $\mathbf{R}_x$ and $\mathbf{R}_y$ tail-to-head.
- Plot the resultant vector **R**.
- Indicate the angle θ between $\mathbf{R}_x$ and **R**.

2. Tail-to-tail method

- Plot a sketch-diagram (i.e. a vector diagram which is not according to scale) with vectors $\mathbf{R}_x$ and $\mathbf{R}_y$ tail-to-tail
- Plot the resultant vector **R**.
- Indicate the angle θ between $\mathbf{R}_x$ and **R**.

*We use **R** when referring to a resultant vector in general. $\mathbf{F}_R$ is used more specifically to refer to a remaining force.*





- **Magnitude of the resultant vector R**

  Utilise **Pythagoras' Theorem** to obtain the **magnitude** of the resultant vector **R**:

$$R^2 = R_x^2 + R_y^2$$

- **Direction of the resultant vector R**

  Utilise **trigonometry** to obtain the **direction** of the resultant vector **R**:

$$\tan\theta = \frac{R_y}{R_x}$$

Describe the direction (θ) of **R** according to the methods you learned to indicate direction

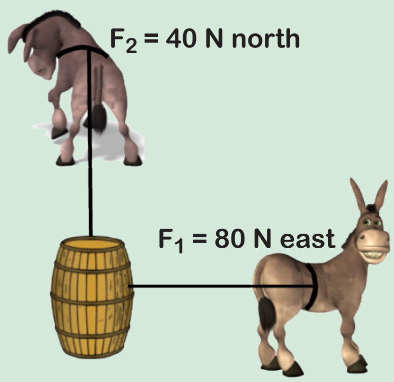
In Grade 11 we use examples which include **force vectors** and **displacement vectors** to graphically calculate the magnitude and direction of the resultant. Let us look at two computed examples:

Example 1 Force vectors

Two forces, $\mathbf{F}_1$ of 80 N east and $\mathbf{F}_2$ of 40 N north are simultaneously exerted on a wooden barrel as illustrated in the sketch.

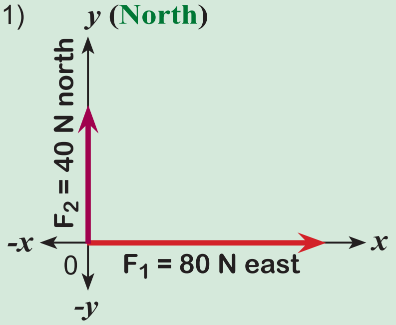
(1) Plot a sketch-diagram (i.e. a vector diagram which is not according to scale) of the force vectors on a Cartesian plane.

(2) Using a scale of 10 mm : 10 N and the tail-to-tail method. Calculate the magnitude and direction of the resultant of these two forces graphically,

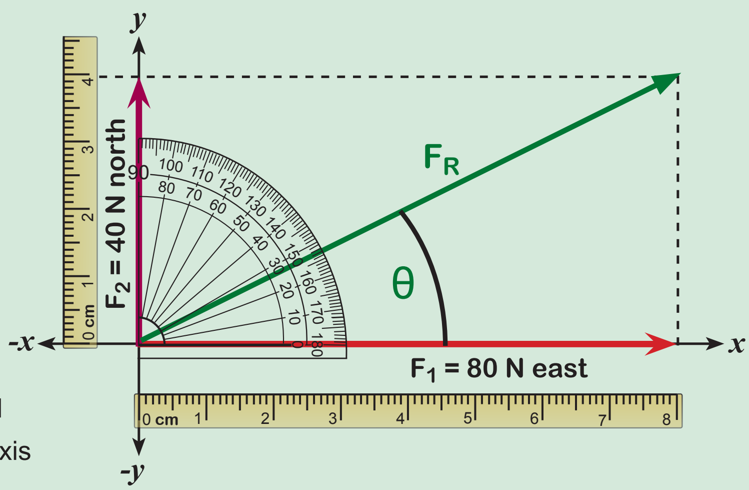


Answer

(1)



(2) **Scale:** 10 mm :10 N
(I.e.: 1 mm:1 N)



Magnitude: F_R = [89 (mm) × 1 N] = **89 N**

Direction: θ = **27°** above the positive x-axis

The magnitude and direction of the resultant force: F_R = **89 N**, **27°** above the positive x-axis
(or, **27°** north of east or E **27°** N)
(or, direction / bearing **63°**)

Example 2 Displacement vectors

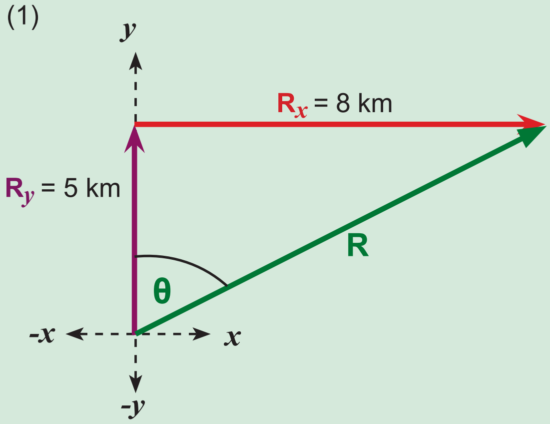
Susan walks 5 km in the direction 0˚ (in the positive y-direction), where after she changes course and then walks 8 km in the direction 90˚ (in the positive x-direction)

(1) Plot a **sketch-diagram** (i.e. a vector diagram not according to scale), to indicate the two displacements Susan underwent, as well as her resultant displacement, using the tail-to-head method. label all information completely in the sketch.

(2) **Calculate** the magnitude and direction of Susan's resultant displacement.

Answer

(1)



(2) Magnitude of **R**

$$\mathbf{R}^2 = \mathbf{R}_y^2 + \mathbf{R}_x^2$$
$$= (5\text{ km})^2 + (8\text{ km})^2$$
$$= 25\text{ km}^2 + 64\text{ km}^2$$
$$= 89\text{ km}^2$$
$$\mathbf{R} = 9{,}4\text{ km}$$

Susan's resultant displacement,
R = 9,4 km direction 58°

(or, **58°** east of north or N **58°** E)
(or, **32°** above the positive x-axis

Direction of **R**

$$\tan\theta = \frac{\mathbf{R}_y}{\mathbf{R}_x}$$
$$= \frac{8\text{ m}}{5\text{ m}}$$
$$= 1{,}6$$
$$\theta = \tan^{-1}(1{,}6)$$
$$\theta = 58°$$

Exercise 2 RESULTANT OF TWO PERPENDICULAR VECTORS

1. Explain the following concepts:

Explain the following concepts : ______________________________

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Perpendicular vectors : ______________________________

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Tail-to-head method : ______________________________

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Tail-to-tail method : ______________________________

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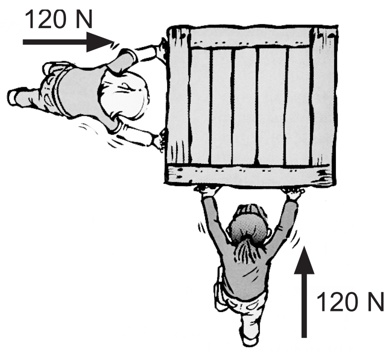
2. Two forces of 120 N each are exerted on a crate simultaneously as shown in the figure.



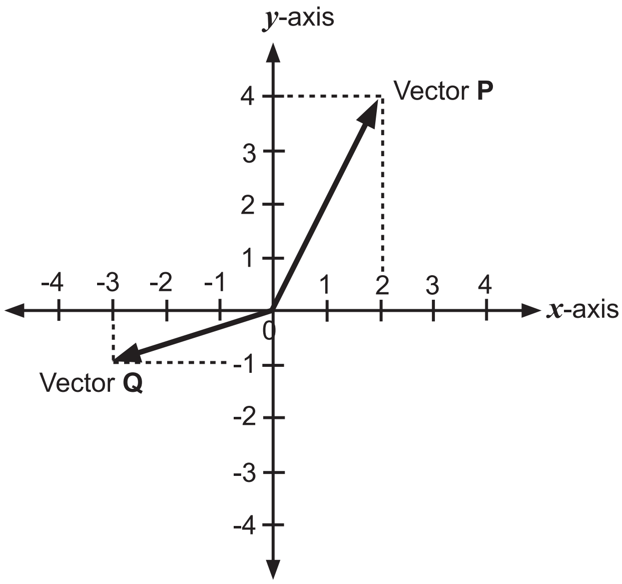
   2.1 Plot a sketch diagram of the force vectors on a Cartesian plane.

   2.2 Using a suitable scale of 10 mm : 20 N and the **tail-to-tail** method determine the magnitude and direction of the resutants of these two forces graphically.

3. Force vectors **P** and **Q** are plotted according to scale on the Cartesian plane shown below..

   Calculate:

   3.1 the magnitude of vector **P**, in force units.



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   3.2 the direction of vector **Q**, measured clockwise from the positive y-axis

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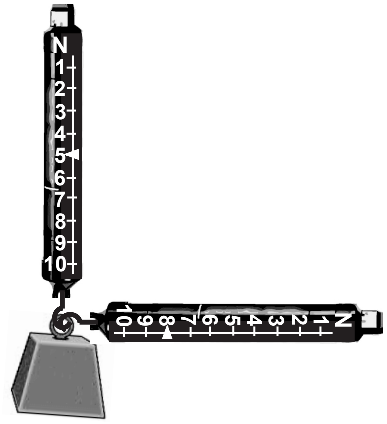
4. Two Newton spring balances are used to exert forces on a wooden block as shown in the figure.

   4.1 Take the readings of the individual forces exerted on the spring balances and plot them on a Cartesian plane.



   4.2 Plot a vector sketch diagram (not according to scale) of the two forces and their resultant.

   4.3 Calculate the magnitude and direction of the resultant of the two forces.

5. A boy on his surfboard moves 150 m due west, after which he changes direction, moving 70 m due north. Plot the two displacements on a Cartesian plane and then calculate the boy's resultant displacement relative to the horizontal plane.

6. While being rowed across a river 800 m wide, a boat is swept 500 m downs team before reaching the opposite bank. Determine the resultant displacement of the boat by means of an accurate scale diagram. Utilise the head-to-tail method.

7. A horizontal and a vertical force is exerted on an object. The horizontal force is established as being 8 N. Graphically determine the magnitude of the vertical force, as well as the angle between the resultant and the horizontal force, if the magnitude of the resultant is 10 N.

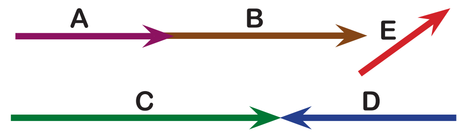
8. A horizontal and a vertical force are exerted on an object. The vertical force is established as being 5 N. Calculate the magnitude of the horizontal force, as well as the angle between the resultant and the horizontal force, if the magnitude of the resultant is 9 N.

THE RESULTANT OF MORE THAN TWO

I. ADDITION OF COLLINEAR VECTORS

Two or more vectors operating in one and the same dimension (i.e. appearing on the same straight line), are called **collinear vectors**. Collinear vectors can be aligned either in the same or in opposite directions. In the accompanying figure the vectors are all collinear vectors, except vector E.

We are now going to learn how to represent collinear horizontal vectors and collinear vertical vectors on a Cartesian plane and how to determine the resultant of these vectors.



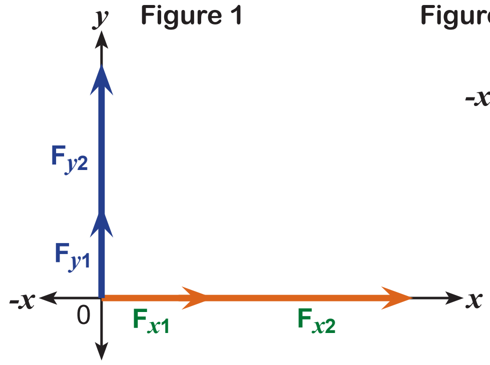
A ,B, C and D are collinear horizontal vectors.

A and B are collinear vectors in the same direction. C and D are collinear vectors in the oppsite direction.

Representation of collinear vectors on the Cartesian plane

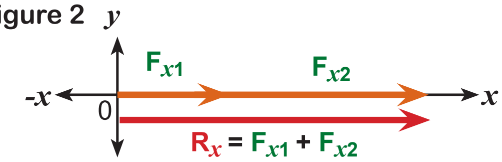
When two or more vectors simultaneously work in the horizontal x-direction, we can plot these vectors on the Cartesian plane with the head-to-tail method; thus they represent the **collinear horizontal vectors**. In **Figure 1** below, F_{x1} and F_{x2} are two **force vectors** (collinear forces), working simultaneously in the horizontal x-direction.

Figure 1



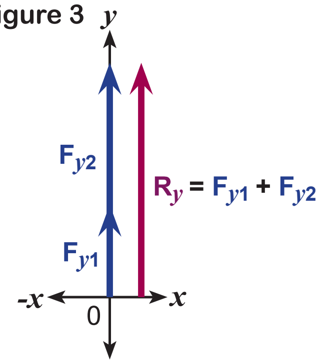
Collinear forces represented on a Cartesian plane.

Figure 2



Net or resultant horizontal force vector R_x.

Figure 3



Net or resultant vertical force vector R_y.

In the same way, when two or more vectors are simultaneously working in the vertical y-direction, we can plot these vectors head-to-tail on the Cartesian plane and thus they represent the **collinear vertical vectors**. In **Figure 1** above, F_{y1} and F_{y2} are two **force vectors** (collinear forces), working simultaneously in the vertical y-direction.

Add the collinears together

The advantage of collinear vectors is that they can be added together algebraically.

Now we join together the collinear horizontal vectors by adding up all vectors in the horizontal direction (i.e. parallel in the x-direction) to determine the **net or resultant horizontal vector R_x**, (also known as the net x-component), as indicated in **Figure 2** above. The net horizontal force vector or net x-component of F_{x1} and F_{x2} then equals:

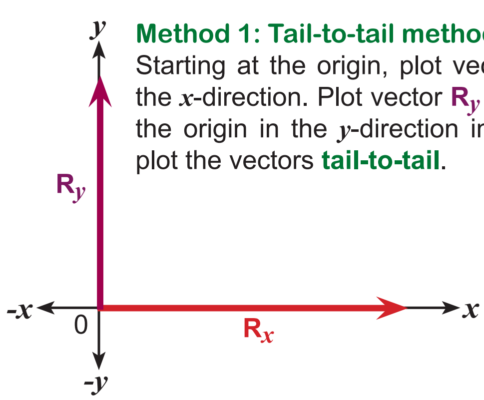
$$R_x = F_{x1} + F_{x2}$$

Likewise we add the vertical collinear vectors together by adding all the vectors in the vertical direction (ie perpendicular to the x-direction) to determine the **net or resultant vertical vector R_y** (also called the net y-component), as in **Figure 3** above. The net vertical force vector of net y-component of the two forces F_{y1} and F_{y2} is:

$$R_y = F_{y1} + F_{y2}$$

Sketch R_x and R_y on the Cartesian plane

After determining the net horizontal vector (net x-component) R_x and the net vertical vector (net y-component) R_y, we can plot these two vectors on a new Cartesian plane, as illustrated in the figure below. There are two methods we can use to plot the two vectors R_x and R_y on the Cartesian plane.

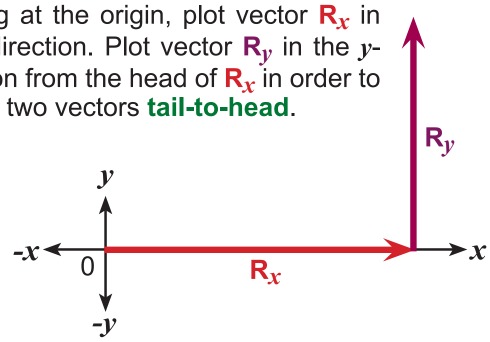


Method 1: Tail-to-tail method

Starting at the origin, plot vector R_x in the x-direction. Plot vector R_y also from the origin in the y-direction in order to plot the vectors **tail-to-tail**.

Method 2: Tail-to-head method

Starting at the origin, plot vector R_x in the x-direction. Plot vector R_y in the y-direction from the head of R_x in order to get the two vectors **tail-to-head**.



Determine the resultant R of R_x and R_y

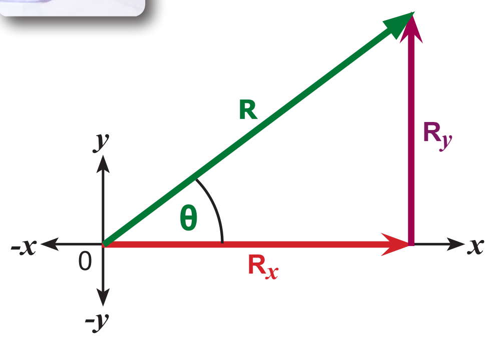
The resultant vector **R** of two or more vectors is the single vector which has the same effect as the original vectors together. There are two methods to determine the resultant vector **R** of two perpendicular vectors R_x and R_y : the **tail-to-head method** and the **tail-to-tail method**.

I. Determining the resultant vector graphically

1. Tail-to-head method

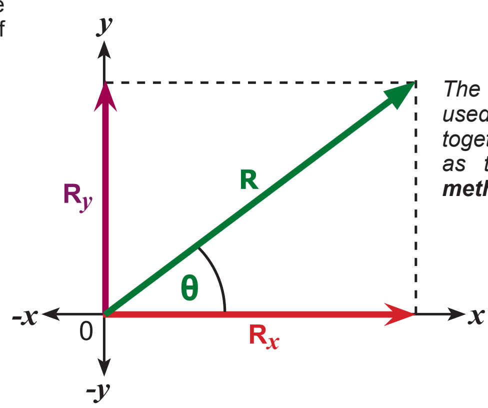
- Obtain a suitable scale, e.g. Scale: 10 mm : 10 N.
- Plot the first vector, R_x according to scale.
- Plot the second vector, R_y according to scale, attaching its tail to the head of R_x
- Connect the tail of R_x with the head of R_y to produce the resultant vector **R**.
- Measure the length of **R** with a ruler and the direction (**θ**) of **R** with a protractor.
- Use the chosen scale to determine the true magnitude of the resultant.
- Describe the direction (**θ**) according to the methods you have learnt to indicate direction.

The tail-to-head method used to add two vectors together, is often referred to as the **triangular method** because the third side of the triangle is formed by the resultant



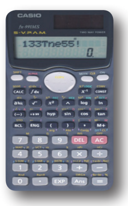
2. Tail-to-tail method

- Obtain a suitable scale, e.g. Scale: 10 mm : 10 N.
- Plot the first vector, R_x according to scale.
- Plot the second vector, R_y according to scale, start at the same origin as R_x (i.e. start at the tail of R_x).
- Complete a rectangle by plotting two lines of equal length parallel to R_x en R_y
- Plot the resultant vector **R** as the diagonal of the rectangle, starting at the tails of R_x and R_y.
- Measure the length of **R** with a ruler and the direction (**θ**) of **R** with a protractor.
- Use the chosen scale to determine the true magnitude of the resultant.
- Describe the direction (**θ**) of **R** according to the methods you have learnt to indicate direction.



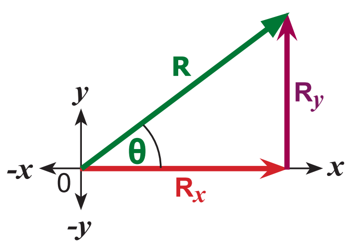
The tail-to-tail method used to add two vectors together, is referred to as the ***parallelogram method****, as it forms a parallelogram*

II. Determining the resultant vector algebraically



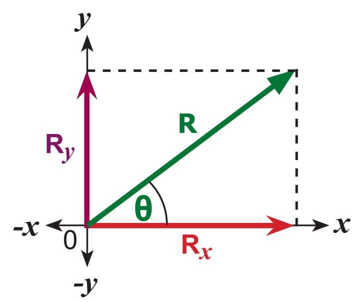
1. Tail-to-head method

- Plot a sketch-diagram (i.e. a vector diagram which is not according to scale) with vectors R_x and R_y tail-to-head.
- Plot the resultant vector **R**.
- Indicate the angle **θ** between R_x and **R**.



2. Tail-to-tail method

- Plot a sketch-diagram (i.e. a vector diagram which is not according to scale) with vectors R_x and R_y tail-to-tail
- Plot the resultant vector **R**.
- Indicate the angle **θ** between R_x and **R**.



- **Magnitude of the resultant vector R**

  Utilise **Pythagoras' Theorem** to obtain the magnitude of resultant **R**:

$$R^2 = R_x^2 + R_y^2$$

- **Direction of the resultant vector R**

  Utilise **trigonometry** to obtain the **direction** of resultant vector **R**:

$$\tan\theta = \frac{R_y}{R_x}$$

Describe the direction (**θ**) of **R** according to the methods you learned to indicate direction.

We now look at the following examples which include **force vectors** and **displacement vectors** to determine the magnitude and direction of the resultant of collinear horizontal vectors and collinear vertical vectors.

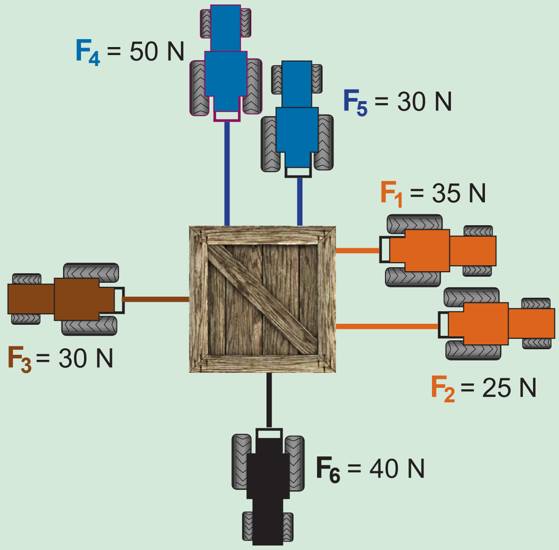
Example 3 Force vectors

The figure shows six toy tractors, all six simultaneously exerting a force on a crate. Three horizontal forces: F_1 = 35 N east, F_2 = 25 N east en F_3 = 30 N west, and three vertical forces: F_4 = 50 N north, F_5 = 30 N north and F_6 = 40 N south.

(1) Plot a sketch diagram of the forces on a Cartesian plane

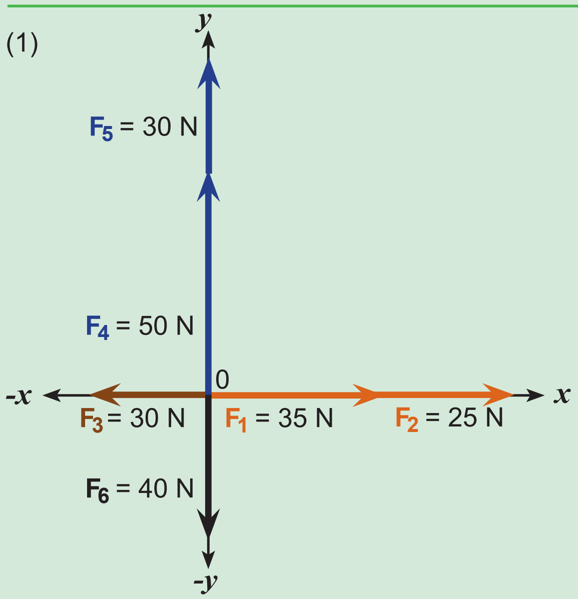
(2) Plot a diagram of the net vertical force (net *y*-component) F_y and the net horizontal force (net *x*-component) F_x. Plot the resultant F_R of F_y and F_x on the diagram. (Use the tail-to-tail method).

(3) Calculate the magnitude and direction of the resultant F_R of the forces.



Answer

(1)



(2)

$F_x = F_1 + F_2 + F_3$

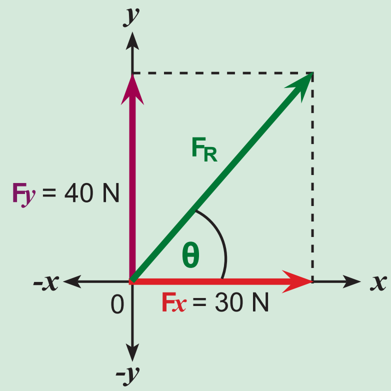
= 35 N + 25 N + (-30 N)

= 30 N east

$F_y = F_4 + F_5 + F_6$

= 50 N + 30 N + (-40 N)

= 40 N north



(3) **Magnitude** of F_R

$F_R^2 = F_y^2 + F_x^2$

$= (40\ \text{N})^2 + (30\ \text{N})^2$

$= 1\,600\ \text{N}^2 + 900\ \text{N}^2$

$= 2\,500\ \text{N}^2$

$F_R = 50$ N

Direction of F_R

$\tan\theta = \dfrac{F_y}{F_x}$

$= \dfrac{40\ \text{N}}{30\ \text{N}}$

$= 1{,}33$

$\theta = \tan^{-1}(1{,}33)$

$\theta = 53{,}1°$

(or direction (bearing of) **36,9°**)
(or E **53,1°** N or N **36,9°** E)
(or, **53,1°** above the positive *x*-axis)

The magnitude and direction of the resultant force: F_R **= 50 N, 53,1° north from east**

Example 4 Displacement vectors

A boy walks 10 m due north, then 3 m due east, stands still for a moment and then walks another 2 m due east, then 4 m due south and a further 13 m due west.

(1) Plot a sketch diagram of the boy's displacement on a Cartesian plane.

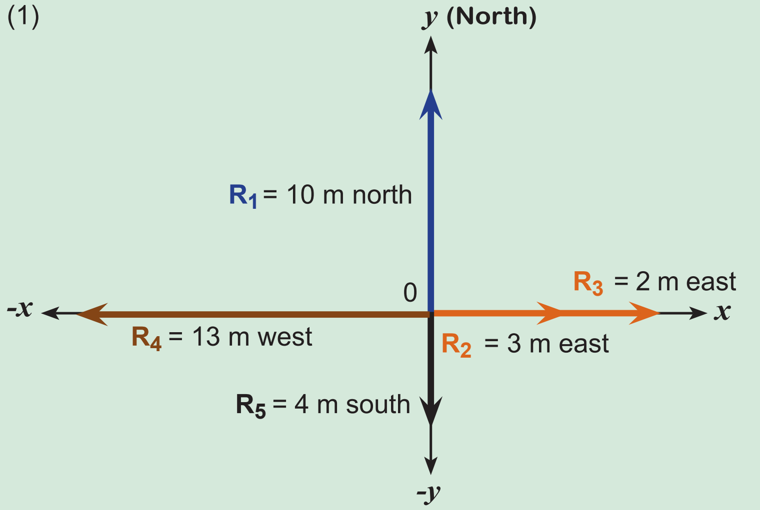
(2) Calculate net displacement (net component) of the boy's displacement in the direction north-south (vertical).

(2) Calculate net displacement (net component) of the boy's displacement in the direction east-west (horizontal).

(3) Plot an accurate vector diagram according to scale (10 mm = 1 m) of the boy's net displacement to the north and his net displacement to the south. Determine the magnitude and direction of his displacement in respect of the point of origin by accurate measurement. (Use the tail-to-head method).

Answer

(1)



(2) *Net displacement vertical*

F_y (or vertical) = R_1 + R_5

= 10 m + (-4 N)

= 6 m north

(3) *Net displacement horizontal*

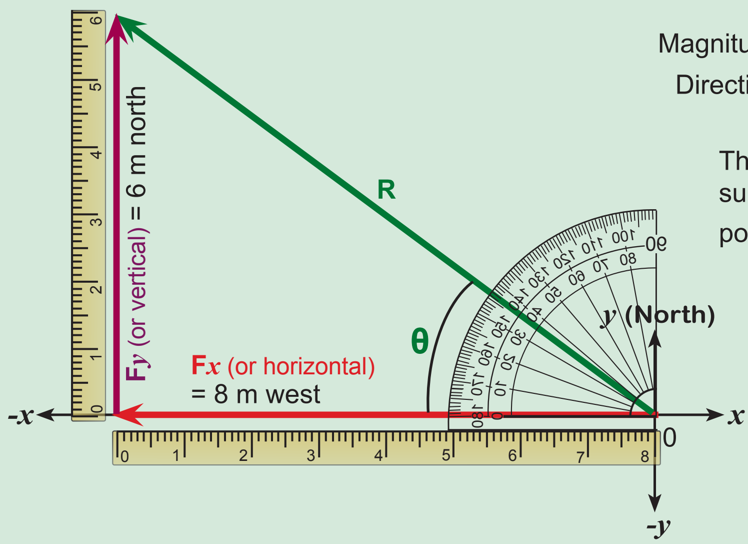
F_x (or horizontal) = R_2 + R_3 + R_4

= 3 m + 2 m +(-13 m)

= -8 m

I.e.: 8 m west

(4) **Scale:** 10 mm = 1 m



Magnitude: **R** = [100 (mm) × 1 m] = **100 m**

Direction: **θ** = **36°** north of west

The magnitude and direction of the resultant displacement from the starting point: **R** = **89 N**, **36°** north of west

(or, W **36°** N or N **54°** W)

(or direction (bearing of) **306°**)

(or, **36°** above the positive x-axis)

Exercise 3 ADDITION OF COLLINEAR VECTORS

1. Explain the concept of collinear vectors.

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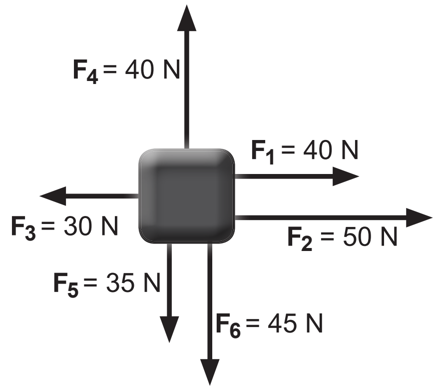
3. The figure shows six forces exercised simultaneously on an object. Three horizontal forces: F_1 = 40 N east, F_2 = 50 N east and F_3 = 30 N east, and three vertical forces: F_4 = 40 N north, F_5 = 35 5 N south and F_6 = 45 N west.

3.1 Plot a sketch diagram of the forces on a Cartesian plane.



2.2 Plot a sketch diagram (not to scale) of the vertical force (net y-component) F_y and the net horizontal force (net x-component) F_x. Draw the resultant F_R of F_y and F_x on the diagram. (Use the tail-to-tail method.)

2.3 Calculate the magnitude and direction of the resultant F_R of the forces.

Magnitude: ______________________________ Direction: ____________________

______________________________ ____________________

______________________________ ____________________

______________________________ ____________________

3. Force vectors F_1, F_2, F_3, F_4, F_5 and F_6 are shown according scale on the Cartesian plane below.

3.1 Calculate the net force (net component) of all the forces in the horizontal x-direction.

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3.2 Calculate the net force (net component) of all the forces in the vertical y-direction.

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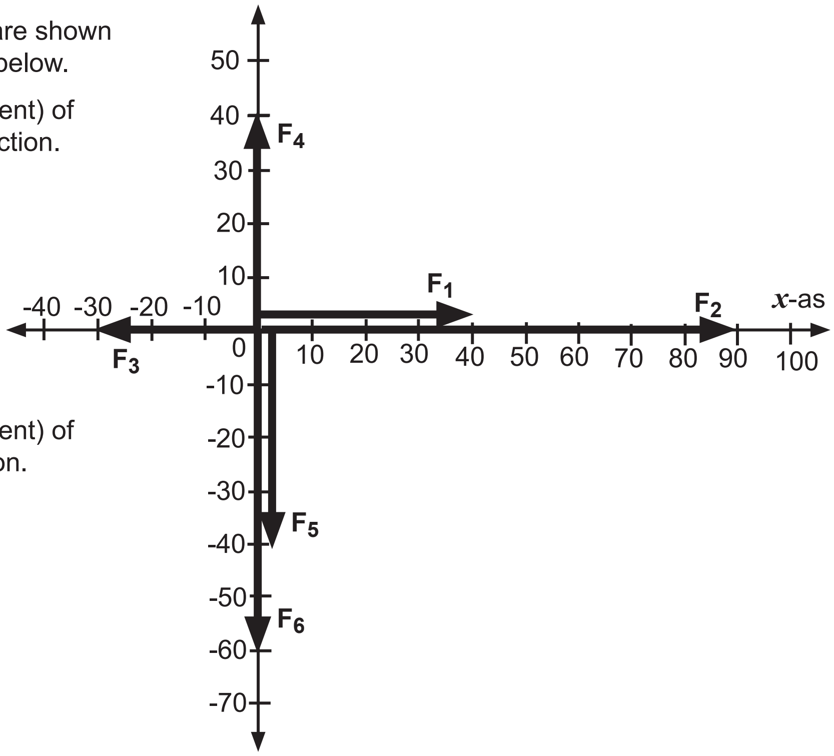


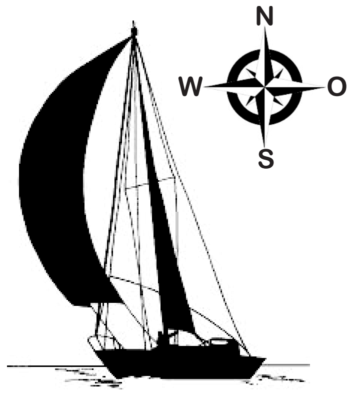
3.3 Plot a sketch diagram of the net horizontal force (net x-component) F_x and the net vertical force (net y-component) F_y. Draw the resultant F_R of F_x and F_y on the diagram. (Use the tail-to-head method.)

3.4 Calculate the magnitude and direction of the resultant F_R of the forces.

Magnitude: ______________________ Direction: ______________________

4. A yacht sailing on the following route from the port.

5 km north, 2 km north, 3 km east, 2 km east and then 4 km south



4.1 Plot a sketch diagram upon a Cartesian plane of the displacement of the yacht

4.2 Calculate net displacement (net component) of the yacht in the north-south direction (vertical)

4.3 Calculate net displacement (net component) of the yacht in the east-west direction (horizontal)

4.4 Plot a accurate vector diagram to scale (10 mm:1 m) of the yacht's net displacement north and its net displacement south. By accurate measurement determine the magnitude and direction of the yacht's displacement with respect to the port (starting point). (Use the tail-to-head method.)

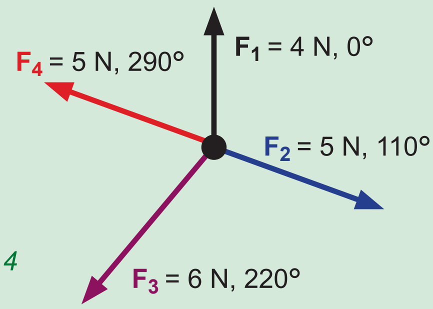
II. ADDITION OF VECTORS WHICH ARE NOT COLLINEAR OR PERPENDICULAR

Occasionally several vectors can exist, e.g. forces acting on an object from different directions or multiple displacements can occur in different directions. The resultant vector of **any** number of vectors on a 2 dimensional plane (2D- plane) is then calculated by the **polygon method of tail-to-head** addition.

- The first arrow is plotted from a suitable starting point to represent one of the vectors according to a suitable scale.
- A second arrow is plotted from its end to represent the second vector.
- This process is repeated until each of the vectors is represented by an arrow.
- The answer will not be influenced by the sequence (consecutive order) of the illustrated vectors.
- The resultant is indicated by the arrow plotted from the starting point (tail) of the first arrow to the end (head) of the last arrow.
- The magnitude and direction of the resultant of the vectors are determined by accurate measurement.
- This method applies to all kinds of vectors (e.g. force, displacement, velocity, etc.). It can also be used to determine the resultant of only two vectors.

Example 5 Force vectors

Four forces, consisting of $\mathbf{F_1}$ = 4 N, $\mathbf{F_2}$ = 5 N, $\mathbf{F_3}$ = 6 N en $\mathbf{F_4}$ = 5 N are exerted at the same point on an object, in directions 0˚, 110˚, 220˚ and 290˚ respectively. Determine the vale and direction of the resultant of the forces with the aid of an accurate scale drawing.

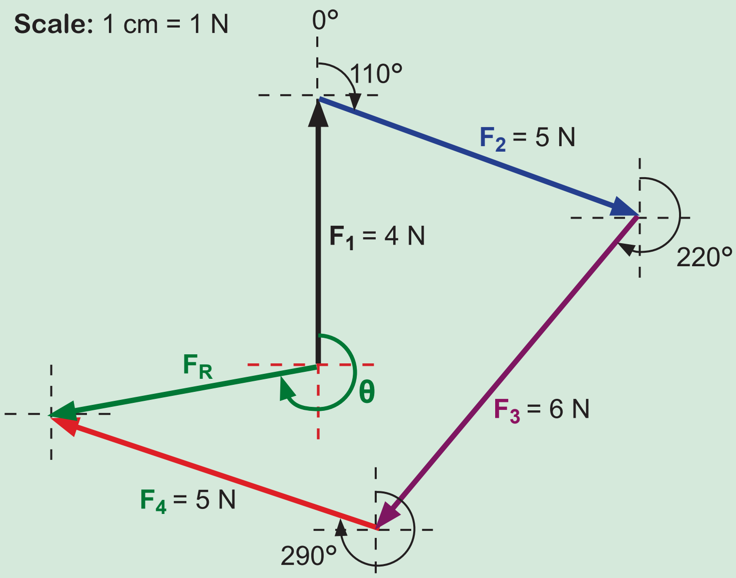


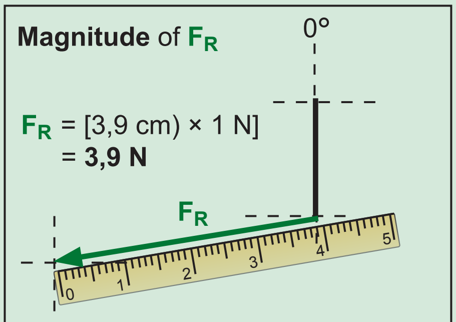
Answer

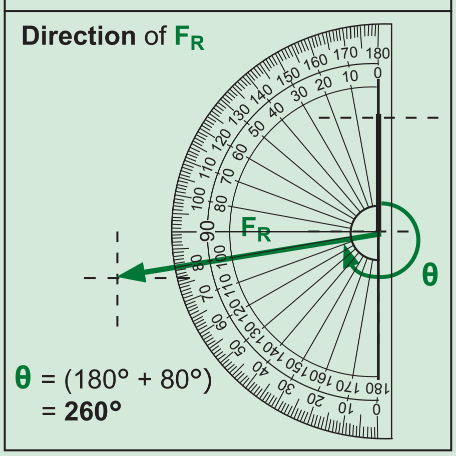
Utilise the **polygon method of tail-to-head addition**.

- *Start with a small axes system and plot according to scale an arrow of 4 cm in the direction 0˚ to represent* $\mathbf{F_1}$ *in magnitude and direction.*
- *Plot a new small axes system at the point (head) of* $\mathbf{F_1}$ *. From this axes system, and according to scale, plot an arrow of 5 cm in the direction 110˚ to represent* $\mathbf{F_2}$ *and both its magnitude and direction.*
- *Plot a new small axes system at the point (head) of* $\mathbf{F_2}$ *. From this axes system, and according to scale, plot an arrow of 6 cm in the direction 220˚ to represent* $\mathbf{F_3}$ *and both its magnitude and direction.*
- *Plot a new small axes system at the point (head) of* $\mathbf{F_3}$ *. From this axes system, and according to scale, plot an arrow of 5 cm in the direction 290˚ to represent* $\mathbf{F_4}$ *and both its magnitude and direction.*
- *Now connect the beginning (tail) of* $\mathbf{F_1}$ *with the end (head) of* $\mathbf{F_4}$ *, with the arrow pointing in the direction of* $\mathbf{F_4}$ *. This arrow (vector) which completes the polygon, is the* ***resultant force*** $\mathbf{F_R}$ *of the four forces exerted on the object.* ($\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} + \mathbf{F_4}$).
- *Determine the magnitude and direction of* $\mathbf{F_R}$ *by accurate measurement.*

Scale: 1 cm = 1 N







The resultant force, $\mathbf{F_R}$ = **3,9 N**, direction **260°**

(*or* ***10°*** *under the negative* x*-axis*)

(*or* ***10°*** *South of West, of W* **10°** *S*)

(*or* ***80°*** *West of South, or S* ***80°*** *W*)

Example 6 Displacement vectors

A boat starts navigating the following route from the harbour:
62 km in the direction 90˚ ; 15 km in the direction 180˚ ;
77 km in the direction 315˚ and then 30 km in the direction 240˚ .
Using a scale of 10 mm = 10 km, determine:

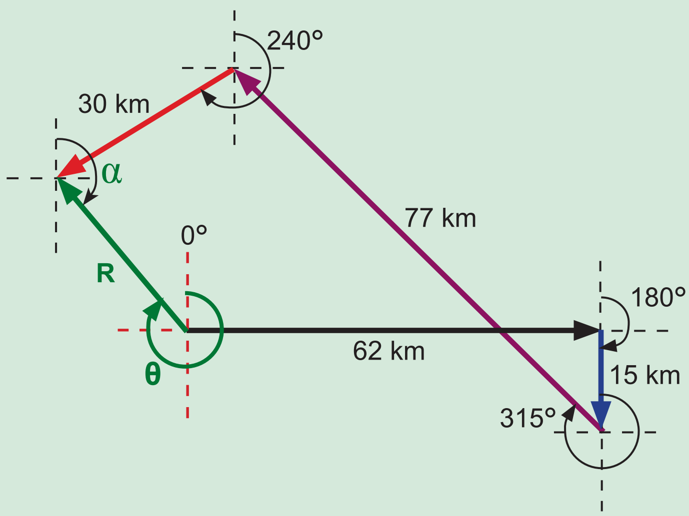
(1) the boat's resultant displacement;
(2) the direction in which the boat has to navigate in a straight line to return to the harbour.

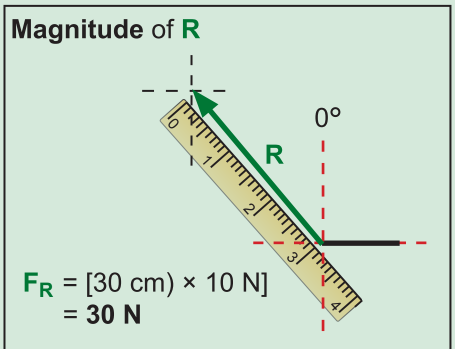
Answer

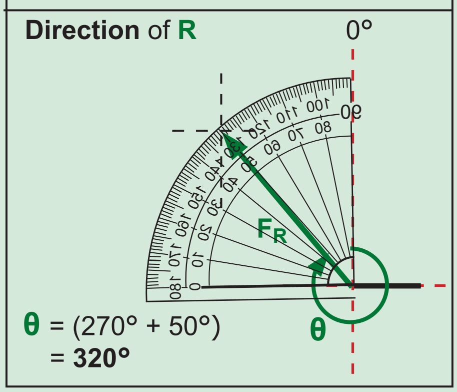
(1) Utilise the **polygon method of tail-to-head addition**

- *Start with a small axes system and, according to scale, and draw an arrow of 6,2 cm (62 mm) in the direction 90˚ to indicate the magnitude and direction of the displacement. Plot a new axes system at the end (head) of the arrow.*
- *From the new axes system, draw an arrow of 1,5 cm (15 mm) in the direction 180˚ according to scale, to represent the displacement of 15 km. Plot a new axes system at the end (head) of the arrow.*
- *From the new axis system, draw an arrow of 7,7 cm (77 mm) in the direction 180˚ according to scale, to represent the displacement of 7 km. Plot a new axes system at the end (head) of the arrow.*
- *From the new axes system, draw an arrow of 3 cm (30 mm) in the direction 240˚ according to scale, to represent the displacement of 30 km.*
- *Now connect the start point (tail) of the first arrow with the end (head) of the last arrow, in the direction of the last arrow. This arrow (vector), which completes the polygon, is the* ***resultant vector R*** *of the four displacements the boat underwent.*
- *Determine the magnitude and direction of* ***R*** *by accurate measurement.*

Scale: 10 mm = 10 km







The resultant displacement, R = **30 km**, direction **320°**

(*or* ***50°*** *above the negative x-axis*)

(*or* ***50°*** *North of West, or W* ***40°*** *N*)

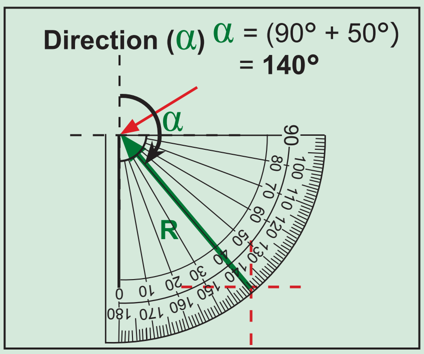
(*or* ***40°*** *West of North, or N* ***40°*** *W*)

(2) Direction to navigate back to the harbour = α = direction **140°**

(*or* ***50°*** *under the negative x-axis*)

(*or* ***50°*** *South of East, of E* ***50°*** *S*)

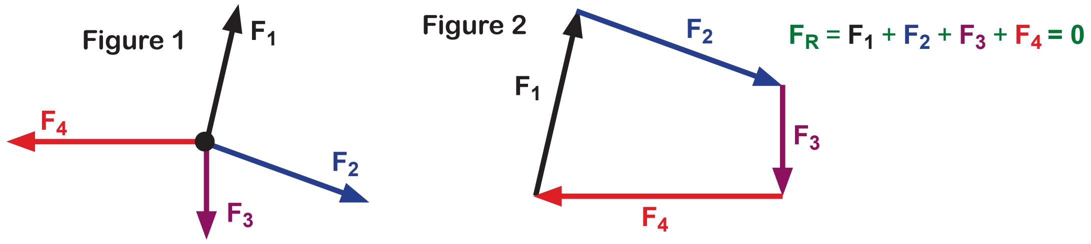
(*or* ***40°*** *East of South, or S* ***40°*** *E*)



Note: If the end of the last displacement coincides with the beginning of the starting point of the first displacement, the **figure is closed**. In such a case it is a **zero resultant** or **zero displacement**. The boat is then **back at he starting point** (harbour).

• Closed polygon of forces

If the result of the vector addition by way of the tail-to-head method is a **closed polygon**, then the **resultant** force **F_R = 0 N and the forces are in equilibrium** (balanced). Such an object will stay at rest or move at a constant velocity.



The four forces in equilibrium is represented in sequence by the sides of a polygon. They form a polygon of forces.

Figure 1 above represents the force vectors of four known forces, exerted on an object simultaneously. The **polygon** in Figure 2 is the vector diagram obtained by placing the vectors tail-to-head, in sequence (in magnitude and direction). The vector diagram is a **closed polygon** which means that the four forces do not have a resultant force.

$F_R = F_1 + F_2 + F_3 + F_4 = 0$

If three or more forces do not constitute a **closed** triangle or polygon, it means that there is a resultant force. The vector that has to be plotted to complete the figure, will represent the resultant force if its direction is not measured in the same sequence. When a triangle or polygon of forces constitute a closed figure with its sides in sequence, the **resultant force is zero**, in other words the **forces** exerted on the same point, **are in equilibrium** (are balanced).

Exercise 4 ADDITION OF VECTORS WHICH ARE NOT COLLINEAR OR PERPENDICULAR

1. Define **resultant of two or more vectors** in words.

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2. Name the method used to graphically determine the resultant of any number of forces (two, three, four or more) acting simultaneously in different directions on an object.

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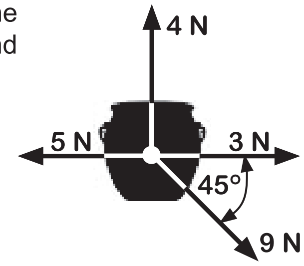
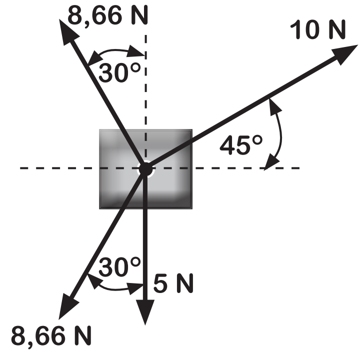
3. Four forces of 80 N, 100 N, 110 N and 160 N act in at the same point (object) into directions of respectively 90 °, 45 °, 300 ° and 250 °. Use an accurate scale drawing where 10 mm = 20 N to determine the resultant (magnitude and direction) of these forces.

4. Forces of 4 N, 3 N, 9 N and 5 N are exerted simultaneously on a object as shown in the figure. Use an accurate scale drawing to determine the resultant force (magnitude and direction) of all the forces acting on the object.



5. Four forces act in at the same object like show in the figure.
   5.1 Use an accurate scale drawing to determine the resultant (magnitude and direction) all the forces acting on the object.



5.2 What does a vector of this nature reveal?

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6. A fisherman in a boat sails at first 4 km south from the port, later he sails 5 km east and then 3 km in the direction 45 ° N of E. Use an accurate scale diagram (where 1 km is represented by 10 mm) in order to determine the following:

   6.1 How far is the angler from the harbor after all the movements?

   6.2 In which direction must he sail in order to get to the port again via a straight line?

7. A long distance runner ran a route that consists of four legs. These are: **10 m east, 16 m in the direction 30 °, 20 m east of 30 ° south and 20 m in the direction 300 °**

   7.1 What is the total distance ran by the athlete? ______________________________

   7.2 Use a scale (10 mm = 2 m) and draw an accurate scale diagram to the represent route the athlete traveled. Determine from your diagram the athlete's resultant displacement.

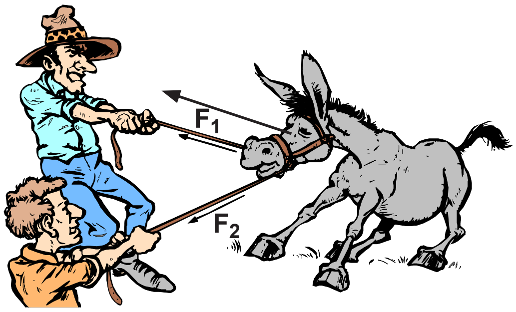
RELATIONSHIP BETWEEN THREE NON-LINEAR FORCES IN EQUILIBRIUM

FORCES IN EQUILIBRIUM

When an object is stationary (or is moving at a constant velocity) it is possible

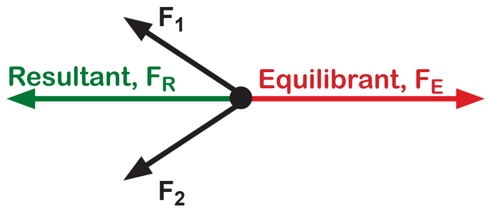
(1) that no forces are exerted on it, or

(2) that the forces exerted on it are **balanced** , in other words the forces are cancelled to deliver a **zero resultant**. We say these cancelled **forces are in equilibrium**.



> ***When the resultant of two or more forces exerted on a point (object) are ZERO, the forces are in equilibrium (balanced).*** $F_R = 0$

In the figure an stubborn donkey are held by Jack and Ronald so that no movement occurs. Two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$, are exerted by the men. The resultant of these forces, $\mathbf{F_R}$, is balanced exactly by a force $\mathbf{F_3}$, which is exerted by the donkey. Force $\mathbf{F_3}$ which **equals** $\mathbf{F_R}$, but is exerted in the **opposite direction** of $\mathbf{F_R}$, is called the **equilibrant** (that which keeps the balance) $\mathbf{F_E}$. $\mathbf{F_E} = -\mathbf{F_R}$



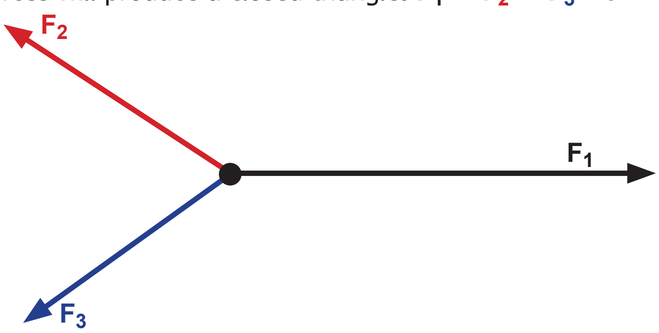
> ***The equilibrant (that which keeps the balance) of two or more forces is the single force which keeps the other forces in equilibrium (balance). The equilibrant has the same magnitude as the resultant of the forces, but operates in the opposite direction.***

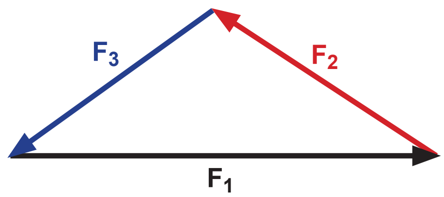
When three forces are exerted at a point (object) and it stays stationary, or it moves at a constant velocity, the forces are in balance and the resultant force equals zero. A head-to-tail diagram of the three forces shows that they form a triangle with no resultant. It is called the **Triangle rule of three forces in Equilibrium** and is defined thus:

> ***When the forces exerted at a point (object) are in equilibrium, their in magnitude and direction can be represented sequentially by the sides of a triangle.***

["Sequentially" is intended to indicate they are plotted "consecutively"]

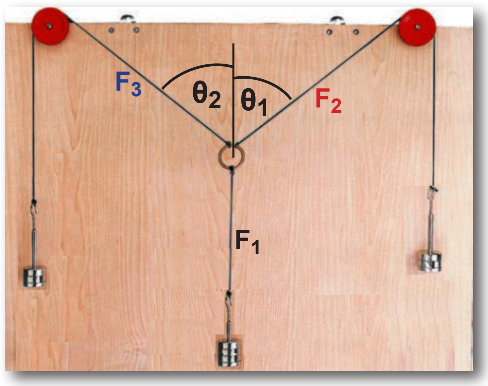
As the three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$, which are exerted on a point, are in equilibrium, a head-to-tail diagram of these forces will produce a closed triangle. $\mathbf{F_1 + F_2 + F_3 = 0}$





RESULTANT OF THREE NON-LINEAR FORCE VECTORS

The figure on the right indicates the setup of an apparatus which is used in an experiment to determine the resultant of three non-linear forces.



Two pulleys are attached to a force board.

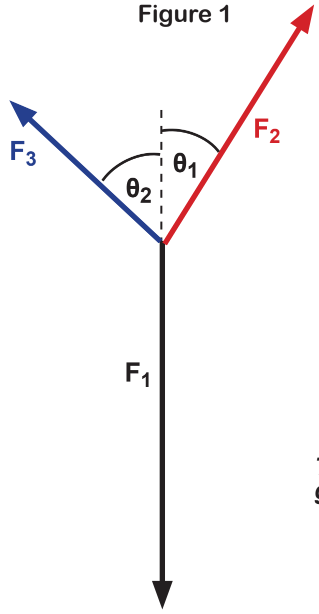
- A sheet of white paper is attached to the board.
- Three pieces of string are tied to a ring.
- Weights are tied to the other end of each string and they are draped over the pulleys in such a way that they can move freely; the ring are then adjusted vertically so that they are in the approximate centre of the sheet of paper.
- The three forces exerted on the ring, are balancing one another (the system is in equilibrium).
- The centre of the ring is plotted on the paper, as are the directions in which forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ are exerted.
- The magnitude of the three forces is determined by calculating the weight of each one in N (newton) with w (F_g) = mg = (mass in kg)(9,8 $m \cdot s^{-2}$).

- The mass of each weight is noted on the paper – i.e. the magnitude of each force.
- The paper is removed from the board. The forces are marked as in figure 1 and the angles θ_1 and θ_2 are measured in relation to the vertical line.
- The tail-to-head method is implemented to calculate the resultant of the three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$.
- We plot the vertical force first, followed tail-to-head by the other two forces. Ensure that the angle from the vertical line to each force is correct.

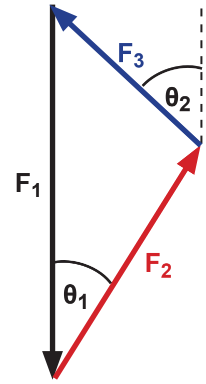
Discussion

- The system of strings and weights is at rest, and the three forces acting on the ring are in balance. The resultant should be zero.
- The tail-to-head vector addition of the three forces constitute a closed triangle (the last vector's head is at the tail of the first vector). The resultant is zero. This confirms that the forces are balanced (forces are in equilibrium).
- It confirms the Triangle rule of three forces in Equilibrium.

Figure 1



Plot the three forces on the paper



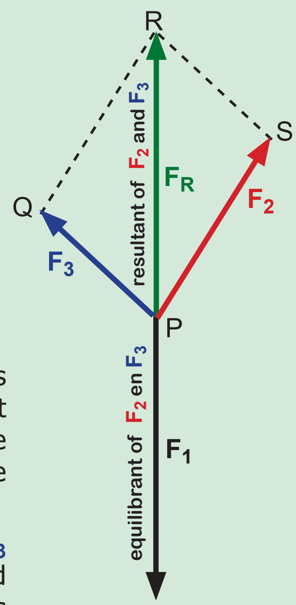
Tail-to-head vector diagram of the three forces

Note! The resultant of TWO vectors can also be determined tail-to-tail with a parallelogram of forces (parallelogram method), which reads as follows:

> ***When TWO forces are exerted on a point, these two forces can be represented in magnitude and direction on the adjacent sides of a parallelogram. The resultant of the forces will then be represented by the diagonal (corner to corner line) of the parallelogram. The equilibrant will be of equal size as the resultant force, but in the opposite direction.***

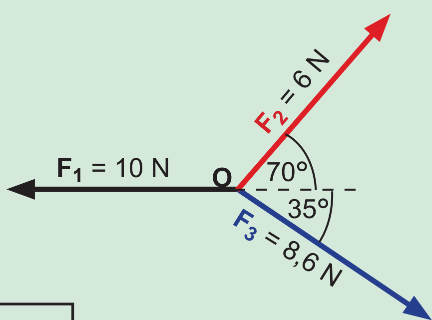
The diagonal (PR) of the completed parallelogram PQRS constitutes the resultant $\mathbf{F_R}$ of the two force vectors $\mathbf{F_1}$ and $\mathbf{F_3}$ .The resultant force equals the third force $\mathbf{F_1}$, but in the opposite direction. Force $\mathbf{F_1}$ therefore, is the equilibrant of $\mathbf{F_2}$ and $\mathbf{F_3}$ and balances these two forces. $\mathbf{F_R} = \mathbf{F_1}$.

Also remember that any one of the three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ can be an equilibrant, because if one of the forces are altered (disturbed), the equilibrium will be disturbed. The equilibrants are any one of the forces keeping the system in equilibrium and balancing the other forces. $\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = \mathbf{0}$.



Example 7

Graphically determine if the three forces exerted on point O are in equilibrium.

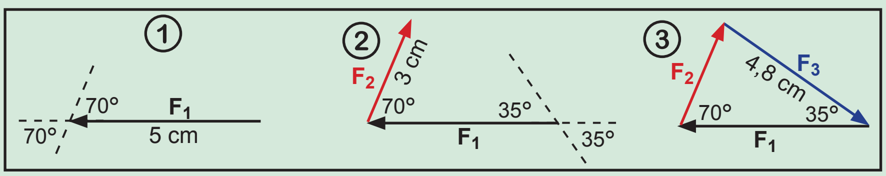


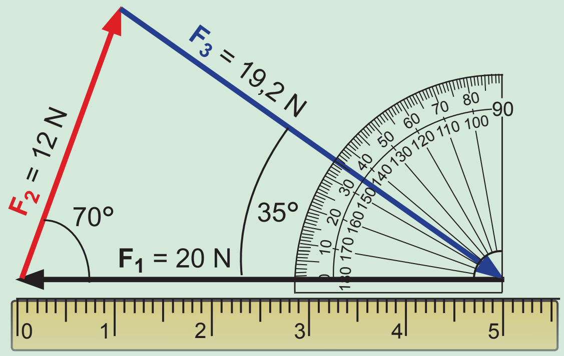
Answer

Scale: 1 cm = 2 N

We plot the three forces tail-to-head. If they jointly form a closed triangle of forces, they are in equilibrium.

Steps to be taken for graphic solution:

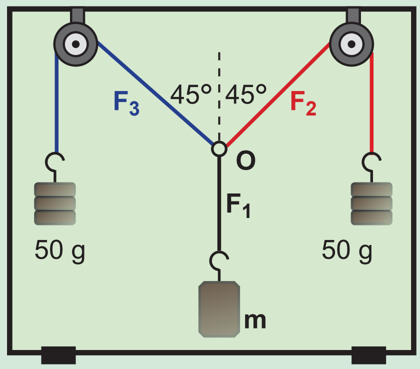




The resultant of the three forces = 0 N
It is a closed triangle of forces, thus the three forces are in equilibrium (balance each other).

Example 8

In an experiment conducted to determine the unknown mass of an object, two 100 g weights and an object of unknown mass (m) are suspended on three strings as illustrated in the diagram (not according to scale). The strings are light and inelastic. Two strings are draped over frictionless pulleys.
When the three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ exerted on ring **O**, are in equilibrium, the angles between the two strings and the vertical line are 45˚ each, as illustrated in the diagram.



(1) Plot a force diagram (diagram of all the forces) of the forces exerted on ring **O**. Indicate the magnitude of each force on the diagram.
(2) By accurate construction and measurement, determine the unknown mass. Use scale 10 mm = 0,1 N.

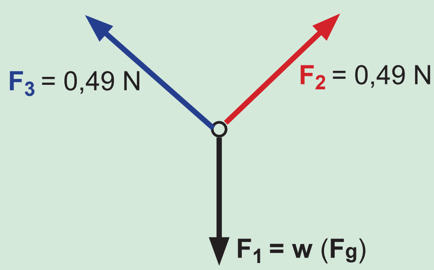
Answer

(1) *A force diagram is an illustration of the object in question, with all the forces exerted on it indicated by arrows.*

- *Before illustrating the diagram we must first determine the magnitude of forces* F_2 *and* F_3*, by calculating the **weight** (**w**) of the weights.*

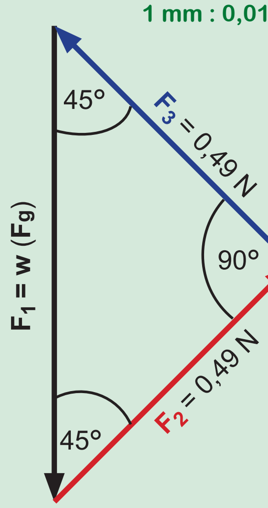
10 = $\frac{15}{1\,000}$ = 0,05 kg

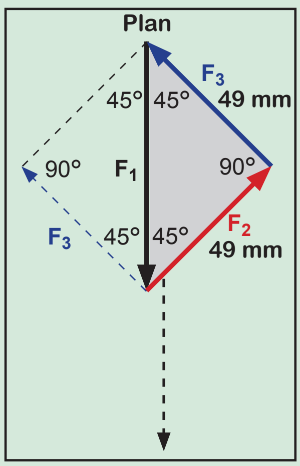
$F_2 = F_3 = w_2$ (of F_{g_2}) $= w_3$ (of F_{g_3}) $= mg$
$= (0,05 \text{ kg})(9,8 \text{ m·s}^{-2})$
$= 0,49$ N



(2) **Scale:** 10 mm = 0,1 N

10 mm : 0,1 N
1 mm : 0,01 N





$F_1 = w\,(F_g)$
$= [69 \text{ (mm)} \times 0,01 \text{ N}]$
$= \mathbf{0,69 \text{ N}}$

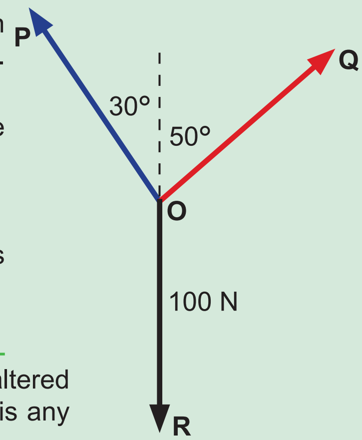
$\mathbf{w = mg}$

$\mathbf{m = \frac{w}{g}}$

$m = \frac{0,69 \text{ N}}{9,8 \text{ m·s}^{-2}}$
$= 0,07 \text{ kg} = (0,07 \times 1\,000)$
$= 70$ g

Example 9

The accompanying sketch illustrates three forces OP, OQ and OR which are in equilibrium. The magnitude of force OR = 100 N and is exerted vertically down.

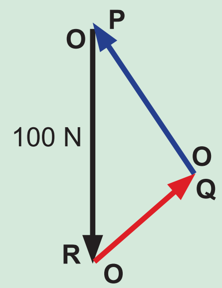


(1) Which of the three forces is the equilibrant force?
(2) Plot a vector diagram representing the magnitude and direction of the three forces.
(3) What is this diagram called?
(4) Write down the magnitude and direction of the resultant of OP and OQ.
(5) Using a scale of 10 mm = 20 N, determine the magnitude of the forces exerted on OP and OQ.

Answer

(1) Any one of the three forces, because if any one of the three forces are altered (disturbed), the equilibrium will be disturbed. Therefore the equilibrant is any force keeping the system in equilibrium.

(2)



(3) Closed triangle of forces.

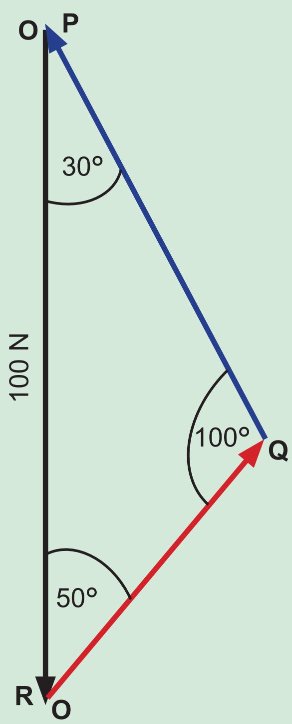
(4) Magnitude = 100 N
Direction = opposite to **OR**.

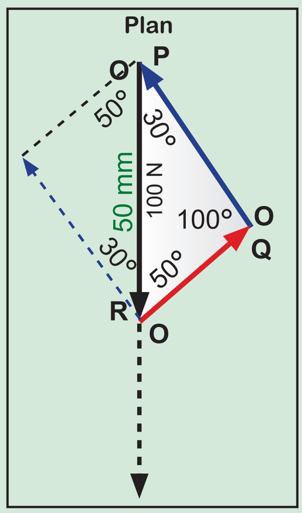
(5) **Scale:** 10 mm = 10 N

10 mm : 10 N
1 mm : 1 N

OP = [77 (mm) × 1 N]
= **77 N**

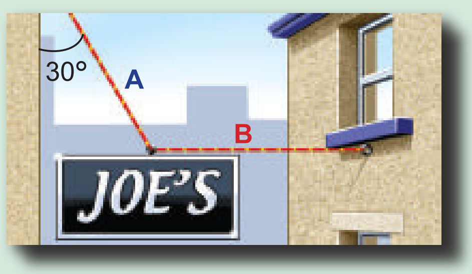
OQ = [50 (mm) × 1 N]
= **50 N**





Example 10

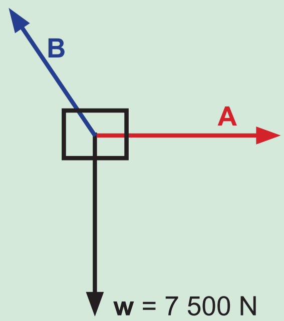
Joe wants to suspend an billboard with a weight of 7 500 N so that cable **A**, which is attached to his shop, forms an angle of 30˚ as illustrated in the figure. Cable **B** is attached to a neighboring building, and is horizontal.



(1) Draw a sketch, indicating the forces exerted on the board, and name each force.
(2) Determine the magnitude of the forces **A** and **B** in the two cables by construction and measurement. (Scale: 10 mm = 1000 N)
(3) If cable **B** is pulled further to enlarge the angle of 30˚, would the force in A ***increase***, ***decrease*** or ***remain constant***? Explain.

Answer

(1)



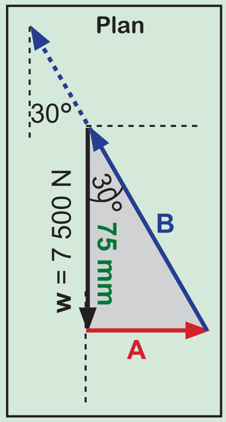
w = weight of board

A = (strain) force in cable

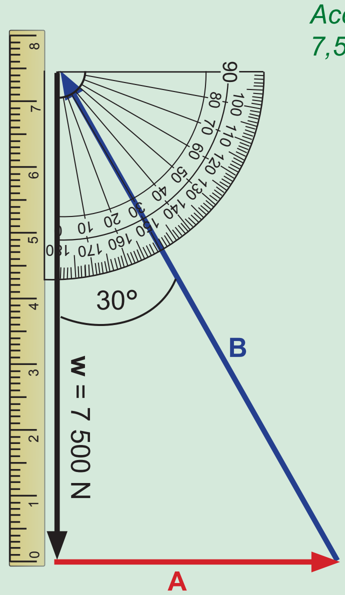
B = (strain) force in cable

$B^2 = w^2 + A^2$

Because the weight **w** = 7 500 N remains the same, the force in **B** will increase if the force in **A** increases so that the angle of 30˚increases.



(2) **Scale:** 10 mm = 1 000 N



According to scale, 75 mm (or 7,5 cm) will represent 7 500 N

A = [(45 mm) × 100 N] = **4 500 N**

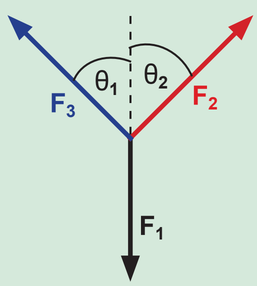
B = [(86 mm) × 100 N] = **8 500 N**

EXSPERIMENT

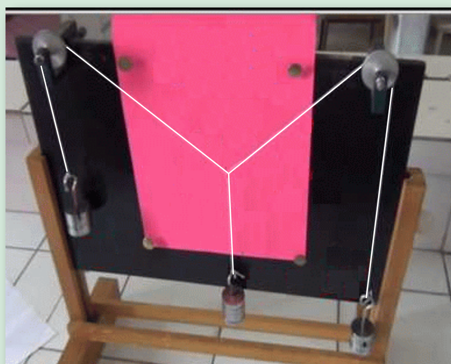
Purpose: Determine the resultant of three non-linear force vectors

What you need
- Force board
- Two pulleys
- String
- Weights
- Hooks/hangers for the weights
- Pencil, paper, ruler and protractor.

Method:
1. Fasten two pulleys on a board. Attach paper to the board as shown in the figure.
2. Put a piece of string over the pulleys and attach a small hook to each end. Tie another piece of string to the first between the pulleys and attach a hook to the other end thereof.
3. Hang suitable weights on the two ends of the original string draped across the pulleys.
4. Place weights on the hook of the middle string so that the knot (where it was tied)comes to rest in front of the paper. Pull the knot slightly out of alignment and let go of it. If the pulleys are reasonably without friction, it should come to rest in the same position.
5. Mark the position of the knot (point of application of the three forces), e.g. point O, and the direction of the strings, i.e. the direction in which the three forces are operating, precisely on the paper. (Watch out for the parallax error!)
6. Remove the paper from the board and plot three vectors of force from point O to represent the three forces. Name them F_1, F_2 and F_3 respectively

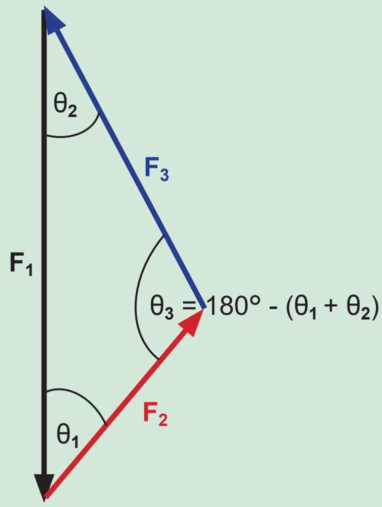


7. Calculate the magnitudes of the three forces by calculating the weight of each weight (weight = mass × 9,8 m·s^{-2} and write it down next to the forces respectively. Remember to include the weight of the hooks. The weights of these three weights represent three non-linear forces.
8. Measure the angles θ_1 and θ_2, and write it on your diagram.



Setup of experiment.

9. Utilise the tail-to-head method to calculate the resultant of the three forces as indicated on the diagram to the right. Ensure each one has the correct angle in relation to the vertical line.



Result:

1. Write down the readings you recorded during the experiment in the following the table and complete the table.

Mass in kg	Force (weight) in N	Magnitude of angle
m_1 =	F_1 =	θ_1 =
m_2 =	F_2 =	θ_2 =
m_3 =	F_3 =	θ_3 =

2. Utilise the data in the table and a suitable scale to determine the resultant of the three forces by means of the tail-to-head method. Ensure each one has the correct angle in relation to the vertical line.

Conclusion:

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Questions:

1. Which method can be used to determine the resultant of forces F_2 and F_3 graphically? What is it also known as?

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2. Which of the forces F_1, F_2 or F_3 is the equilibrant? Explain.

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3. What relation exists between e.g. the resultant of forces F_2, F_3 and their equilibrant?

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Exercise 5 RELATIONSHIP BETWEEN THREE NON-LINEAR FORCES IN EQUILIBRIUM

1. In which instance are two or more **forces in equilibrium**?

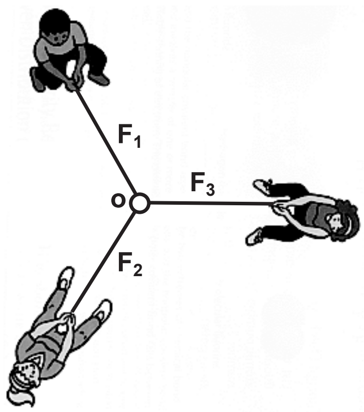
2. How does an object react when the forces acting on the object are in **equilibrium** (balanced)?

3. 3.1 What is a **equilibrant force** (balancing force)?

   3.2 How does it compare with its corresponding resultant force?

4. Force's $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ are exerted on the point (**O**) , as such that they remain in **equilibrium**.

   4.1 Which of these forces are the equilibrant force? Explain.



   4.2 Which of these forces are the resultant force of forces $\mathbf{F_1}$ and $\mathbf{F_2}$?

   4.3 What will happen with the resultant and equilibrant when the angle between $\mathbf{F_1}$ and $\mathbf{F_2}$ is made larger?

   4.4 When are the resultant of $\mathbf{F_1}$ and $\mathbf{F_2}$, the greatest?

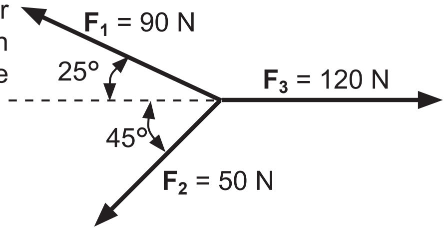
   4.5 When are the resultant of $\mathbf{F_1}$ and $\mathbf{F_2}$, the least?

   4.6 What kind of figure will be obtained when forces F1, F2 and F3 are placed tail-to-head?

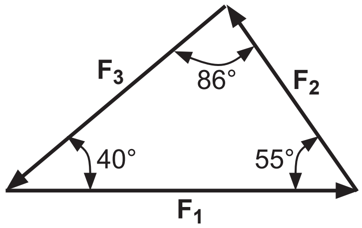
   4.7 Which rule (or law) is applicable here? ____________________

   4.8 Set this rule (law) in question 4.7 above in words.

4.9 Now, using the rule (law) in 4.7 and 4.8 determine graphically whether the three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ which are exerted on point O remain in equilibrium. The magnitude of the forces and the angles between the forces are shown in the figure here.



5. The accompanying diagram shows a tail-to-head vector diagram of three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ which are exerted on an object.



5.1 What can you say about the resultant force that the object experience? Explain

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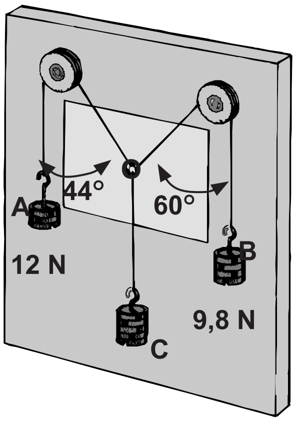
5.2 Make a sketch to show how these forces will act on the object. All angles must be clearly indicated.

5.2 If $\mathbf{F_1}$ = 11 N and $\mathbf{F_2}$ = 7 N determine the magnitude of $\mathbf{F_3}$. (Scale: 10 mm = 5 N)

6. The diagram shows the result obtained in an "equilibrium of forces" experiment. Weights A and B are suspended over frictionless pulleys and held in equilibrium by an unknown weight, C, which hangs vertically downwards.



6.1 Define the term equilibrium as used in this experiment.

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6.2 Write down the main experimental error that can occur during the execution of this experiment.

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6.3 Draw a FORCE DIAGRAM indicating all the forces acting at point O. Indicate the magnitude of each force on the diagram

6.4 Determine carefully through construction and measuring the weight of C. (Note that you need to use a scale of which 5 mm represents 1 force unit for this construction)

6.5 Calculate the mass of C.

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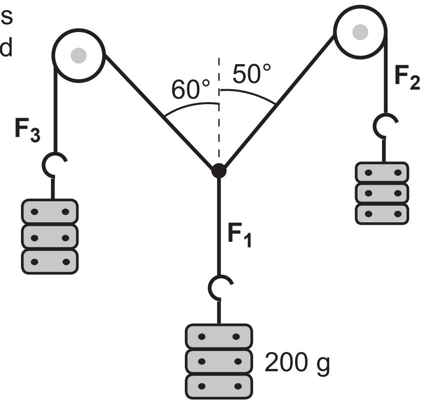
7. Three forces F_1, F_2 and F_3 are connected to two light strings over frictionless pulleys as shown in the diagram. The masses are in equilibrium. F_1, F_2 and F_3 symbolizes the forces in each string respectively.



7.1 Which one of the forces is the **equilibrant force**? Explain.

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7.2 Calculate the **resultant** (magnitude and direction) of F_2 and F_3.

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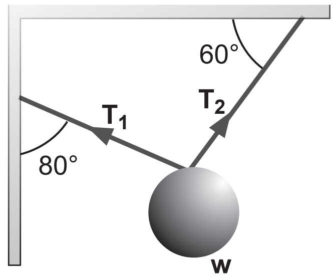
7.3 What is the magnitude and direction of **equilibrant** of F_2 and F_3.

____

7.4 Determine graphically (scale: 10 mm = 0.2 A) the magnitude of forces F_2 and F_3.

8. A metal ball with weight **w** is supported by two ropes fixed to a wall and the ceiling as in the figure. The tension in the various ropes are $\mathbf{T_1}$ and $\mathbf{T_2}$. Tension force $\mathbf{T_2}$ amounted to 80 N



8.1 Draw a force diagram to represent this scenario in terms of "three forces in equilibrium". Show $\mathbf{T_1}$,$\mathbf{T_2}$ and **w** clearly.

8.2 Determine by calculation the weight **w** of the object and the tension $\mathbf{T_1}$.

8.3 Which rule did you apply in Question 6.2? ______________________________

8.4 Which one $\mathbf{T_1}$, $\mathbf{T_2}$ or **w** is the equilibrant? Explain

____

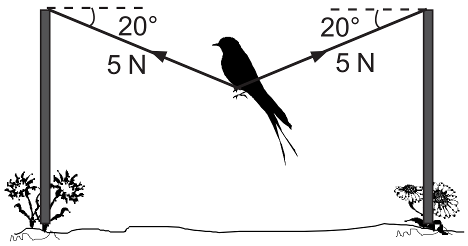
____

8.5 What is the magnitude and direction of the **resultant** of $\mathbf{T_1}$ and $\mathbf{T_2}$? ______________________________

Something to know:

- *Take a look at your triangle of forces in Question 8.2. $\mathbf{T_2}$ will always experience a greater force than $\mathbf{T_1}$, regardless of the size of **w**, and will therefore be the first to break, because the angle opposite (80°) $\mathbf{T_2}$> the angle opposite (30 °) $\mathbf{T_1}$.*
  I.e. tension $\mathbf{T_2}$> $\mathbf{T_1}$.
- *If the angle between 30° T2 and **w** increases, the tentional force increases in $\mathbf{T_1}$.*

9. A bird sits at the center of a thin clothesline as shown in the diagram. This causes the line to bag down at an angle of 20 ° and causes forces in the line of 5N each.



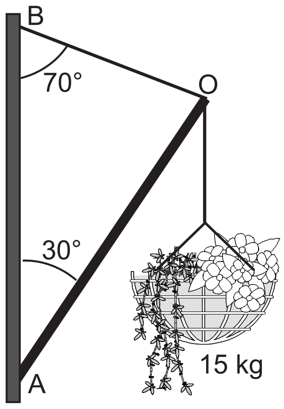
9.1 Consider the small particles at point P under the bird's claws. Draw a diagram showing all the forces acting on the section.

9.2 Point P is in equilibrium. What does this say to us about the forces that act on this point?

____

9.3 Determine using construction the bird's **mass**. (Scale: 20 mm = 1 N)

10. A pot with a mass of 15 kg is suspended by a system consisting of a light weight beam which hinge at A, a lightweight rope attached to O and another rope from O to the wall at B, as shown in the diagram. The angles are shown.



10.1 Calculate the weight of the pot.

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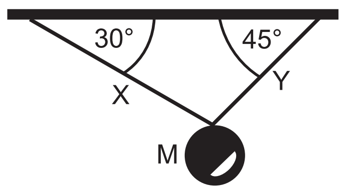
____

____

10.2 Draw a diagram to show all the forces acting on O. Label the forces.

10.3 Determine by an accurate scale drawing the magnitude and direction of (1) the force exerted by the beam on the ring and (2) the tension in the rope to the wall.

11. Two ropes X and Y, to the ceiling tied, supports an object M, as in the diagram. The maximum force that each of these ropes can withstand is 600 N. The weight of M is gradually increased.



11.1 Draw a rough diagram of the triangle of forces and explain by using it which rope will break first.

11.2 Which rope, X or Y, has the greatest amount of tension? Explain your answer.

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____

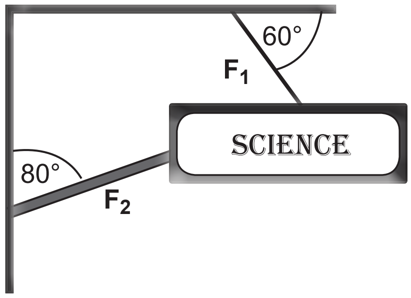
11.3 Determine by means of a scale drawing (10 mm N = 100) the maximum weight of M that can by supported

12. A large billboard is held in position by a cable $\mathbf{F_1}$ with a tensile strength of 800 N and a wooden prop $\mathbf{F_2}$ with breaking a strength of 400 N.

12.1 Draw a triangle of forces to represent equilibrium. Show all forces and angles.

12.2 Which will break first, the cable or the wooden prop if the billboard's mass increases? (HINT: Determine at which values of w will $\mathbf{F_1}$ and $\mathbf{F_2}$ break respectively.)

12.3 What is the maximum weight that the billboard may have?



12.1

12.2 ______________________________

____

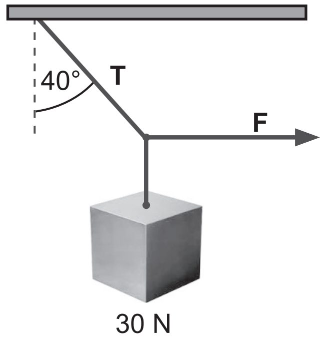
____

____

12.3

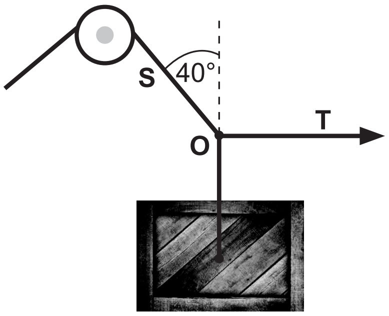
13. An object, weight 30 N, is suspended on a string T, with a horizontal force F pulled to the right, so that it is in equilibrium, as shown in the diagram. The string makes an angle of 40 ° with the vertical.

13.1 Draw a dot to indicate the point O on the diagram, and draw and label all the forces acting at this point. Show the angles clearly.



13.2 Use a scale drawing (10 mm N = 5) to determine the tension T in the string and the force F.

14. A heavy crate is lifted using a rope **S** and a reel so that it can go through a window on the first floor of a building. A second string **T** is drawn horizontally to the left so that the crate doesn't bump against the wall. Mealtime the rope is tied to the crate to suspend in equilibrium, as in the diagram. The tension in the string S is 6500 N.



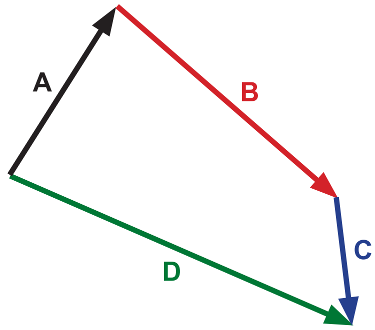
14.1 Draw the point O. Show and label all the forces acting at point O at equilibrium.

14.2 Determine graphically and through the necessary calculations (1) the tension T in the rope and (2) the mass of the crate. (Let 10 mm = 1000 N)

RESOLUTION OF A VECTOR INTO PERPENDICULAR COMPONENTS

I. COMPONENTS OF A VECTOR

We have learnt that two or more vectors, e.g. forces, are replaced by a single vector, namely the resultant. The opposite is also possible: A single vector can be resolved and replaced by two or more separate vectors in such a manner that the resultant of these separate vectors equals the original vector. These separate vectors are called the **components** of the original vector. The components can be in any direction.



> ***The component of a vector is two or more vectors which together have the same effect as the original vector (resultant). Components can be in any direction***
>
> *...OR...*
>
> ***The component of a vector together with another component forms the original vector (resultant). Components can be in any direction.***

The figure above represents four vectors namely **A**, **B**, **C** and **D**, where **D** = **A** + **B** + **C**. The vectors **A**, **B** and **C** can now be regarded as the components of vector **D**.

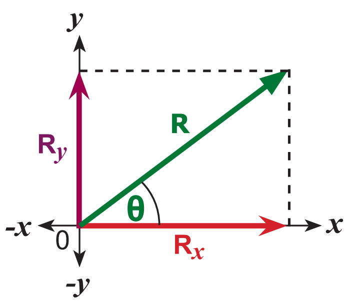
Usually a vector (in a plane) resolves in only two components, of which the direction is perpendicular to each other. Usually one component horizontally or parallel to the x-axis and the other component is usually vertically or parallel to the y-axis.

II. RESOLUTION OF A VECTOR IN ITS HORIZONTAL AND VERTICAL COMPONENTS

Often it is useful to split up an original vector in its components which are perpendicular to each other. This process is called **resolution of a vector into components.** We can then consider the effect of each component separately. The most important case is where we resolves a vector into its perpendicular components. We choose perpendicular components because they operate independently and at an angle of 90°. A change in one will have no effect on the others.

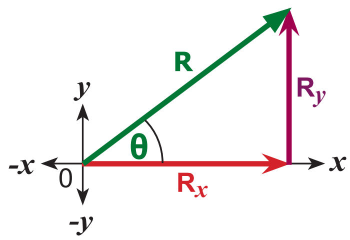
> ***A single vector can be replaced or resolves into two separate vectors, in such a manner that one separate vector is horizontal and one vertical, with the single vector constituting the resultant of the horizontal and vertical vectors.***

Consider a vector **R**, acting at an angle **θ** with the horizontal plane (i.e. at an angle **θ** with the x-axis of a Cartesian plane).The horizontal component R_x and the vertical component R_y of vector **R** are indicated in the following diagrams.



Horizontal and vertical components of R plotted tail-to-tail.

...of...



Horizontal and vertical components of R plotted tail-to-head.

The components R_x and R_y have the same effect as vector **R**. They are perpendicular (90°) towards each other and a change in one has no influence on the other.

By utilizing trigonometric functions we can deduce equations to calculate the components.

- **The horizontal component or component parallel to the x-axis:**

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{R_x}{R}$$

*Thus the **horizontal component** of* **R**:

$$R_x = R\cos\theta$$

- **The vertical component or component parallel to the y-axis:**

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{R_y}{R}$$

*Thus the **horizontal component** of* **R**:

$$R_y = R\sin\theta$$

These two components form the sides of a rectangular triangle (or rectangle) with **R** as diagonal. Out of this it also follows that the magnitude and direction of **R** is related to its components by the equations:

$$R^2 = R_x^2 + R_y^2$$

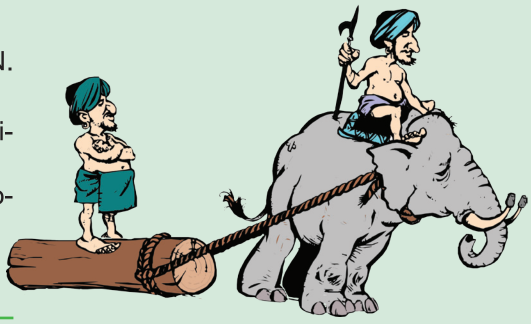
$$\tan\theta = \frac{R_y}{R_x}$$

Example 11 Force vectors

An elephant drags a tree trunk with a force of magnitude 5 000 N. The rope forms an angle of 35˚ with the horizontal plane.

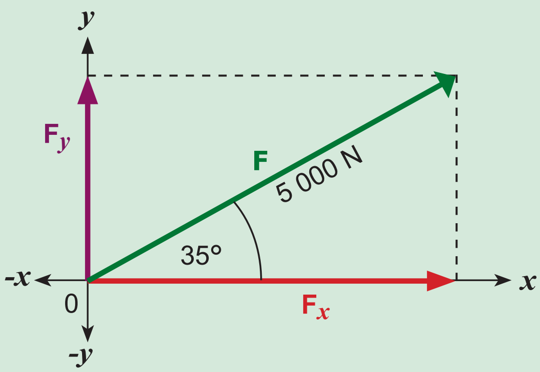
(1) Plot a diagram with captions to indicate the force and its horizontal and vertical components.

(2) Calculate the magnitude of the horizontal and vertical components of the force exerted on the trunk.



Answer

(1)



(2) *Horizontal component* F_x

$F_x = F\cos\theta$
$= (5\ 000\ \text{N})\cos 35°$
$= (5\ 000\ \text{N})(0{,}819)$
$= 4\ 095\ \text{N}$

Vertical component F_y

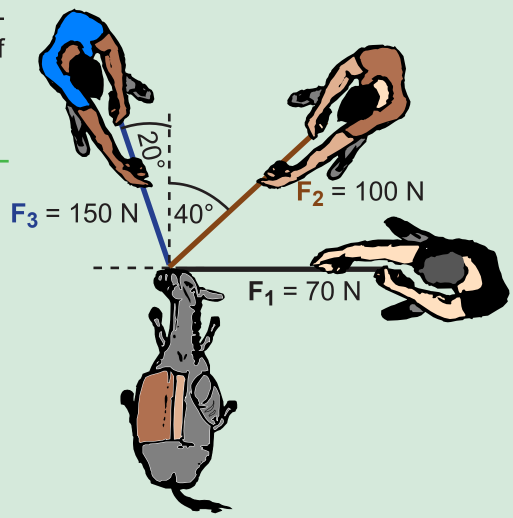
$F_y = F\sin\theta$
$= (5\ 000\ \text{N})\sin 35°$
$= (5\ 000\ \text{N})(0{,}574)$
$= 2\ 870\ \text{N}$

III. ADDITION OF MORE THAN TWO VECTORS

If we want to add together more than two vectors, we can resolve each vector into its horizontal and vertical components. Then we can add up all the horizontal components of all the vectors to obtain the resultant horizontal component. We do the same with the vertical components. Lastly we determine the resultant of the calculated horizontal and vertical components. That gives us the resultant of all the original vectors.

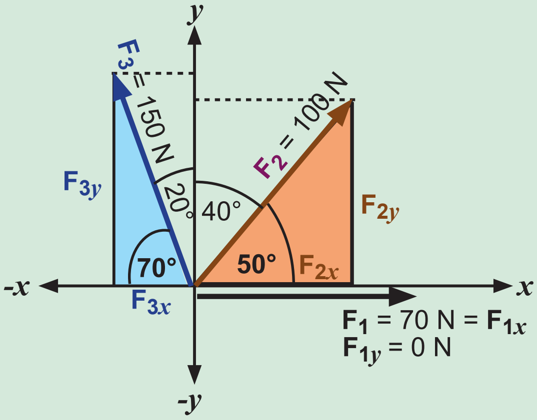
Example 12 Force vectors

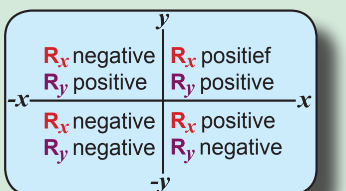
Forces 70 N, 100 N and 150 N are exerted respectively on a donkey with ropes, as indicated in the figure. Calculate the resultant of these forces.



Answer

Step 1 : *Plot each force on a Cartesian plane and indicate each force's x-component, y-component and angle that each one forms with the x-axis.*





The sign (+ or -) of the components of a vector depends on the quadrant in which the vector finds itself..

Step 2 : *Resolve each force into an x-component*

$F_{1x} = F_1 = +70\ \text{N}$

$F_{2x} = +F_2\cos\theta_2 = +(100\ \text{N})\cos 50° = +64{,}28\ \text{N}$

$F_{3x} = -F_3\cos\theta_3 = -(150\ \text{N})\cos 70° = -51{,}3\ \text{N}$

Step 3 : *Resolve each force into an y-component*

$F_{1y} = 0$ N

$F_{2y} = +F_2 \sin\theta_2 = +(100\text{ N})\sin 50° = +76{,}6$ N

$F_{3y} = +F_3 \sin\theta_3 = +(150\text{ N})\sin 70° = +140{,}95$ N

Step 4 : *Determine the resultant x-component*

F_{Rx} (of ΣF_x) = $F_{1x} + F_{2x} + F_{3x}$

= (+70 N) + (+64,28 N) + (-51,3 N)

= 82,98 N

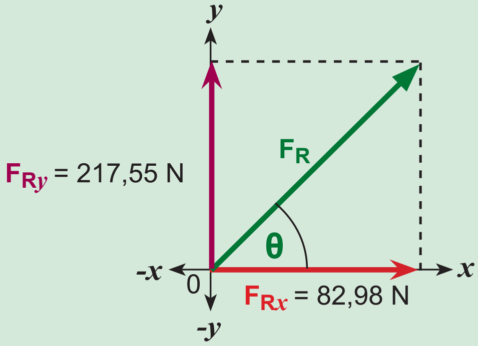
Stap 5 : *Determine the resultant y-component*

F_{Ry} (of ΣF_y) = $F_{1y} + F_{2y} + F_{3y}$

= 0 N + (+76,6 N) + (+140,95 N)

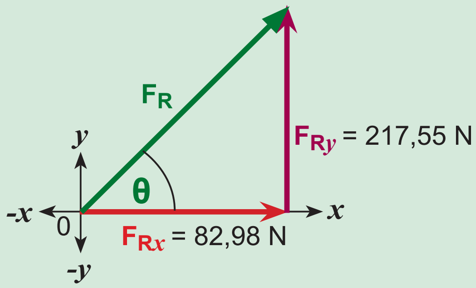
= 217,55 N

Step 6 : *Sketch F_{Rx} and F_{Ry} on a new Cartesian plane and indicate their resultant F_R*



Horizontal and vertical components of F_R plotted tail-to-tail

...of...



Horizontal and vertical components of F_R plotted tail-to-head

Step 7 : *Calculate the **magnitude** of the resultant, F_R of F_{Rx} and F_{Ry}*

$F_R^2 = F_{Rx}^2 + F_{Ry}^2$

$= (82{,}98\text{ N})^2 + (217{,}55\text{ N})^2$

$= 6\,885{,}68\text{ N}^2 + 47\,328\text{ N}^2$

$F_R^2 = 54\,213{,}68\text{ N}^2$

$F_R = 232{,}84$ N

Step 8 : *Calculate the **direction** of the resultant, F_R of F_{Rx} and F_{Ry}*

$\tan\theta = \dfrac{F_{Ry}}{F_{Rx}}$

$= \dfrac{217{,}55\text{ N}}{82{,}98\text{ N}}$

= 2,622

$\theta = \tan^{-1}(2{,}622)$

$\theta = 69.12°$

The positive sign indicates that the angle θ is above the positive x-axis.

The angle will also be positive for an angle θ underneath the negative x-axis.

Step 9 : *Answer*

The magnitude and direction of the resultant force:

F_R = 232,84 N, 69.12° North from East

(or direction **20,88°** (= 90° - 69.12°)
(or E **69.12°** N of N **20,88°** E)
(or, **69.12°** above the positive x-axis

Example 13 Force vectors in equilibrium

Thandi has a painting which she wants to mount on a wall. She uses a piece of fishing line, tied to the corners of the painting to hang it on a hook. The fishing line forms an angle of 20˚ with the top of the painting, and the mass of the painting as well as the fishing line are 8 kg.

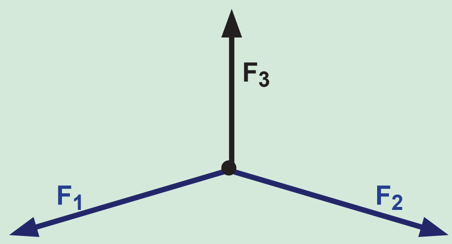
(1) Calculate the weight of the painting (i.e. the gravitational force exerted on the painting by the earth).
(2) At the hook there are three cooperating forces in equilibrium. Plot a diagram with captions of these forces; say what they are.
(3) For the painting to remain hanging, the wall has to exert an upward force on the hook. What is the magnitude and direction of the force, and give an explanation for your answer.
(4) Calculate the force in the fishing line when painting and fishing line are hanging from a hook in the wall.

Answer

(1) **w** (of $\mathbf{F_g}$) **= mg**
= (8 kg)(9,8 m·s^{-2})
= 78,4 N downward

(3) 78,4 N upwards.
The downward weight of 78,4 N equals the upward force exerted by the hook. The weight of the painting is the resultant force of F_1 and F_2, and F_3 the equilibrant of F_1 and F_2 .

(2)



F_3 = upwards force exerted on hook by wall
T_1 = force exerted on hook by fishing line
T_2 = force exerted on hook by fishing line

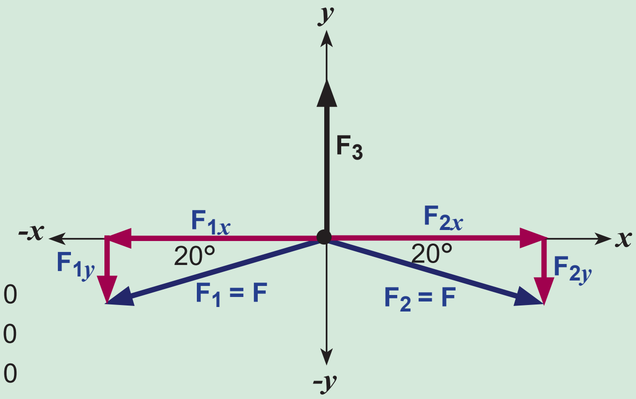
(4) *Because the painting and fishing line are in equilibrium, the sum of the horizontal components of the three forces = 0, and the sum of the vertical components of the three forces = 0.*

Sum of the horizontal components:

F_{R_x} (of ΣF_x) = $F_{1x} + F_{2x} + F_{3x} = 0$
= (-F cos θ) + (+F cos θ) + 0 N = 0
= -F cos 20° + F cos 20° = 0
= 0 N

Sum of the vertical components:

F_{R_y} (of ΣF_y) = $F_{1y} + F_{2y} + F_{3y} = 0$
(-F sin θ) + (-F sin θ) + 78,4 N = 0
-F sin 20° - F sin 20° + 78,4 N = 0
-0,34F -0,34F + 78,4 N = 0
-0,68F + 78,4 N = 0
-0,68F = -78,4 N
F = 115,29 N

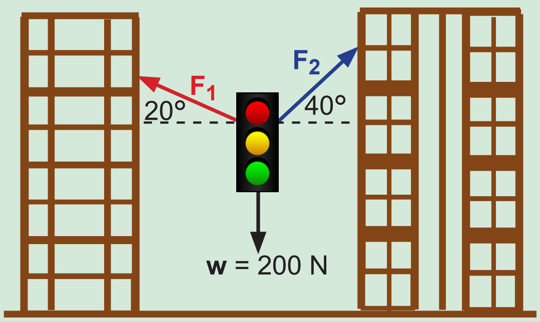


The force in each half of the fishing line is 115.29 N the total force in the fishing line = **230.58 N**
(Note that we have considered the fishing line as two parts, each with a force F)

Example 14 Force vectors in equilibrium

A traffic light with a weight of 200 N is suspended from two adjoining buildings by two cables as indicated.

Calculate the forces F_1 and F_2 in the cables.

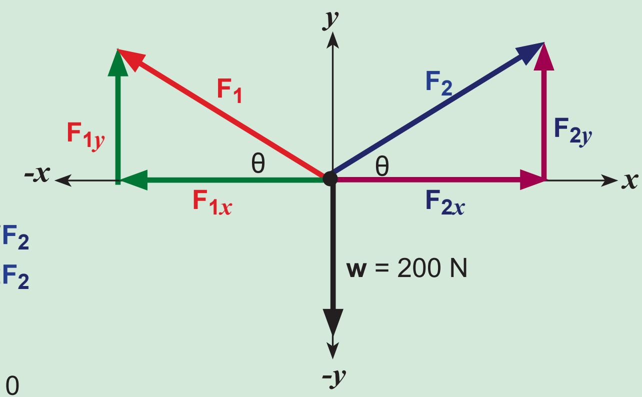


Answer

Because the system is in equilibrium, the sum of the horizontal components of the three forces = 0 and the sum of the vertical components of the three forces = 0.

Sum of the horizontal components:

F_{R_x} (or ΣF_x) = $F_{1x} + F_{2x} + w_x = 0$
(-F_1 cos θ) + (+F_2 cos θ) + 0 N = 0
-F_1 cos 20° + F_2 cos 40° = 0
-0,94F_1 + 0,77F_2 = 0
0,94F_1 = 0,77F_2
F_1 = 0,82F_2



Sum of the vertical components:

F_{R_y} (or ΣF_y) = $F_{1y} + F_{2y} + w_y = 0$
(+F_1 sin θ) + (+F_2 sin θ) + (-200 N) = 0
+F_1 sin 20° + F_2 sin 40° - 200 N = 0
+0,34F_1 + 0,64F_2 - 200 N = 0
200 N = 0,34F_1 + 0,64F_2

Now substitute $F_1 = 0{,}82F_2$ into this equation and calculate F_2.

$200\ N = 0{,}34F_1 + 0{,}64F_2$

$200\ N = 0{,}34(0{,}82F_2) + 0{,}64F_2$

$200\ N = 0{,}28F_2 + 0{,}64F_2$

$200\ N = 0{,}92F_2$

$0{,}92F_2 = 200\ N$

$F_2 = 217{,}39\ N$

We can now use the answer to F_2 and calculate F_1 in:

$F_1 = 0{,}82F_2$

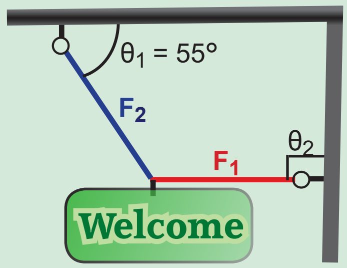
$= 0{,}82(217{,}39\ N)$

$= 187{,}26\ N$

The force F_1 in cable 1 is **187,26 N** and the force F_2 in cable 2 = **217,68 N**.

Example 15 Force vectors in equilibrium

A shop's advertising board exerting a downward gravitational force of 245 N is being supported as shown in the figure.
Calculate the magnitude of forces F_1 and F_2 exerted at the point where the board is supported.



Answer

Given, F_g *(or* **w***) = 245 N,* $\theta_1 = 55°$ *and* $\theta_2 = 90°$*. Asked:* F_1 and F_2

Because the system is in equilibrium, the sum of the horizontal components of the three forces = 0 and the sum of the vertical components of the three forces = 0.

Sum of the horizontal components:

F_{Rx} (of ΣF_x) = $F_{1x} + F_{2x} + F_{gx} = 0$

$(+F_1) + (-F_2 \cos\theta_1) + 0\ N = 0$

$F_1 - F_2 \cos 55° = 0$

$F_1 = F_2 \cos 55°$

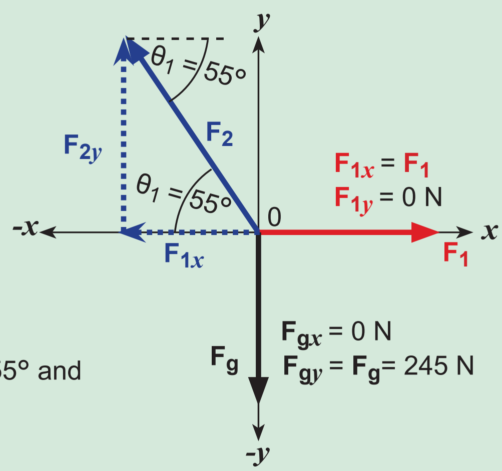
Sum of the vertical components:

F_{Ry} (of ΣF_y) = $F_{1y} + F_{2y} + F_{gy} = 0$

$0\ N + (-245\ N) + (-F_2 \sin\theta_1) = 0$

$F_2 \sin 55° = 245\ N$

$F_2 = \mathbf{299{,}1\ N}$



Substitute now the value of F_2 into the equation of $F_1 = F_2 \cos 55°$ and calculate F_2.

$F_1 = F_2 \cos 55°$

$= (299{,}1\ N) \cos 55°$

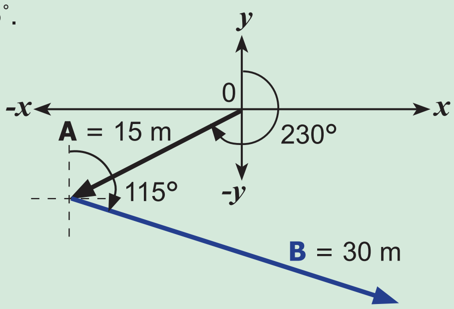
$= \mathbf{171{,}6\ N}$

F_1 = **171,6 N** in the direction of the positive x-axis (to the right)

F_2 = **299,1 N** 55° above the negative x-axis

Example 16 Displacement vectors

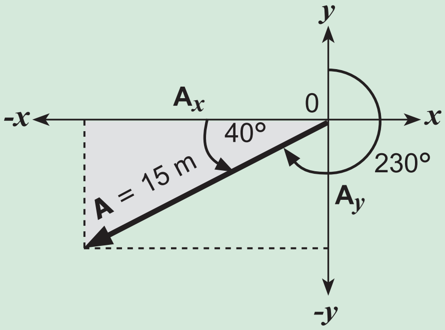
Utilise components to determine the resultant displacement of an athlete who runs 15 m in direction 230˚ and then 30 m in direction 115˚.



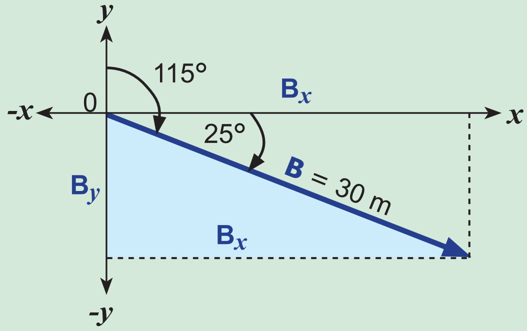
Answer

Step 1 : *Plot each displacement vector* ***on its own*** *Cartesian plane and indicate the x-component, y-component and indicate the angle it forms with the x-axis*

A = 15 m direction 230°



B = 30 m direction 115°



Step 2 : *Resolve each displacement into an x-component*

$\mathbf{A_x} = \mathbf{-A\cos\theta_1} = \mathbf{-}(15\ \text{m})\mathbf{\cos}40° = \mathbf{-}11{,}49\ \text{m}$

$\mathbf{B_x} = \mathbf{+B\cos\theta_2} = \mathbf{+}(30\ \text{m})\mathbf{\cos}25° = \mathbf{+}27{,}19\ \text{m}$

Step 3 : *Resolve each diplacement into an y-component*

$\mathbf{A_y} = \mathbf{-A\sin\theta_1} = \mathbf{-}(15\ \text{m})\mathbf{\sin}40° = \mathbf{-}9{,}64\ \text{m}$

$\mathbf{B_y} = \mathbf{-B\sin\theta_2} = \mathbf{-}(30\ \text{m})\mathbf{\sin}25° = \mathbf{-}12{,}68\ \text{m}$

Step 4 : *Determine the resultant x-component*

$\mathbf{R_x} = \mathbf{A_x} + \mathbf{B_x}$

$= (-11{,}49\ \text{m}) + (+27{,}19\ \text{m})$

$= +15{,}7\ \text{m}$

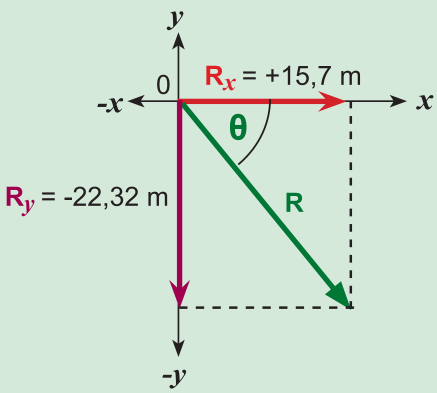
Stap 5 : *Determine the resultant y-component*

$\mathbf{R_y} = \mathbf{A_y} + \mathbf{B_y}$

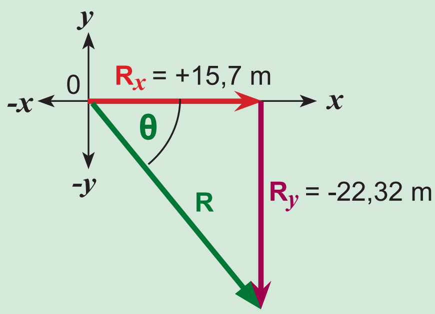
$= (-9{,}64\ \text{m}) + (-12{,}68\ \text{m})$

$= -22{,}32\ \text{N}$

Step 6 : *Sketch* $\mathbf{R_x}$ *and* $\mathbf{R_x}$ *on a new Cartesian plane and indicate their resultant* **R**



Horizontal and vertical components of R plotted tail-to-tail.



Horizontal and vertical components of R plotted tail-to-head.

Step 7 : *Calculate the* ***magnitude*** *of the resultants,* **R** *of* $\mathbf{R_x}$ *and* $\mathbf{R_y}$

$\mathbf{R^2} = \mathbf{R_x^2} + \mathbf{R_y^2}$

$= (+15{,}7\ \text{m})^2 + (-22{,}32\ \text{m})^2$

$= 246{,}49\ \text{m}^2 + 498{,}18\ \text{m}^2$

$\mathbf{R^2} = 744{,}67\ \text{m}^2$

$\mathbf{R} = 27{,}29\ \text{m}$

Step 8 : *Calculate the* ***direction*** *of the resultants,* $\mathbf{F_R}$ *of* $\mathbf{F_{Rx}}$ *and* $\mathbf{F_{Rx}}$

$\mathbf{\tan\theta} = \dfrac{\mathbf{R_y}}{\mathbf{R_x}}$

$= \dfrac{-22{,}32\ \text{m}}{+15{,}7\ \text{m}}$

$= -1{,}422$

$\boldsymbol{\theta} = \mathbf{\tan^{-1}}(-1{,}422)$

$\boldsymbol{\theta} = -54{,}88°$

The negative sign indicates that the angle $\boldsymbol{\theta}$ *is underneath the positive x-axis.*
The angle will also be negative for an angle $\boldsymbol{\theta}$ *above the negative x-axis.*

Step 9 : *Answer*

The magnitude and direction of the athlete's resultant displacement is:

R = 27,59 N, direction (90° + 54,88°) = **144,88°**

(or **54,88°** South of East)

(or E **54,88°** S or S **35.12°** E)

(or, **54,88°** underneath the positive x-axis)

Exercise 6 DECOMPOSITION OF A VECTOR INTO PERPENDICULAR COMPONENTS

1. Define the component of a vector.

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2. What do you understand by the decomposition of a vector into its horizontal and vertical components?

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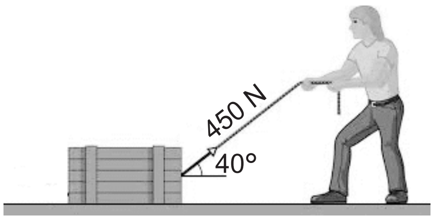
3. A man drags a crate with a force that has a magnitude of 450 N at a **constant speed** over a rugged road. The rope makes an angle of 40 ° with the road.



3.1 Draw a sketch of the crate, show and label all the forces acting on the crate.

3.2 What is the net force acting on the crate? Explain.

3.3 Calculate the magnitude of the vertical upward force with which the man tend to lift the crate from the earth (Ie the vertical component of the applied force).

3.4 Calculate the size of the horizontal frictional force exerted on the crate by gravitation

3.1

3.2 ____

____

3.3 ______________________ 3.4 ______________________

______________________ ______________________

______________________ ______________________

______________________ ______________________

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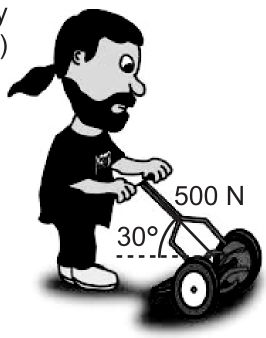
4. A gardener pushes a lawn mower with a mass of 40 kg on a straight lawn. He exerts a force of 500 N on the handle of the mower which is set at an angle of 30 ° with the horizontal, as shown in the figure.

4.1 Calculate the magnitude of the horizontal component of the force exercised by gardener on the lawn mower (Ie the force that tends to move the mower forward.)



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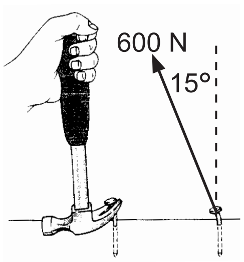
4.2 Calculate the magnitude of the vertical component of the force exercised by the gardener on the lawn mower. (Ie the force that tends push the lawn mower into the earth.)

4.3 Calculate the effective weight of the lawn mower (ie the vertical net downward force on the mower) when pushed.

4.4 Show further that it will be easier for the gardener to pull the lawn mower than pushing it with the same force and angle as shown in the figure.

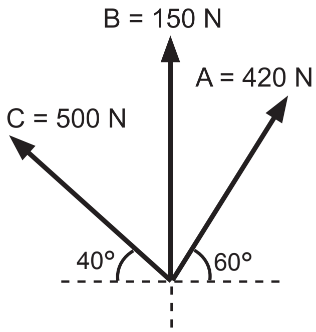
5. A hammer exerts a force of 600 N on a nail at an angle of 15 ° with the vertical, as shown in the figure. Calculate the upward component of the force exerted on the nail.



6. Three forces acting on an object as shown in the figure.

6.1 Calculate the resultant (net) x-component and resultant (net) y-component of these forces.

x-component	y-component
A_x =	A_y =
B_x =	B_y =
C_x =	C_y =
F_{Rx} (of **ΣF_x**) **=**	**F_{Ry}** (of **ΣF_y**) **=**

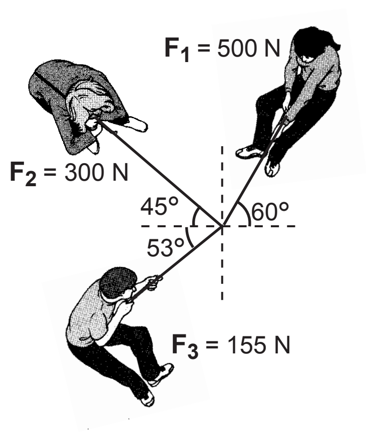


6.2 Sketch **F_{Rx}** and **F_{Ry}** on a new Cartesian plane and show their resultant **F_R**.

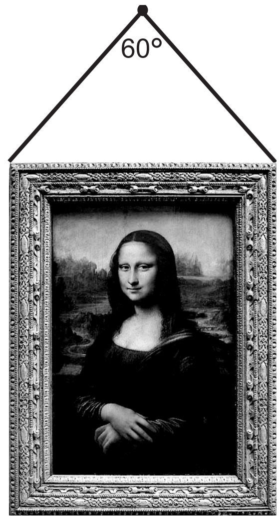
6.3 Calculate the magnitude and direction of the resultant of **F_R**, **F_{Rx}** and **F_{Ry}**.

7. Three forces are applied by three children on an object, exerted as shown in the figure. Calculate the **magnitude and direction** of the resultant of the forces.

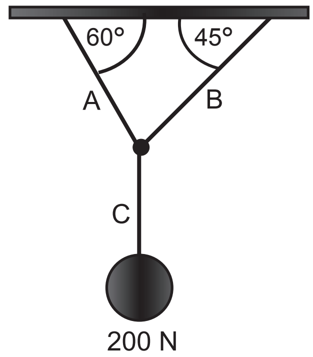
x-component	y-component
F_{1x} =	**F_{1y}** =
F_{2x} =	**F_{2y}** =
F_{3x}	**F_{3y}** =
F_{Rx} (of **ΣF_x**) **=**	**F_{Ry}** (of **ΣF_y**) **=**



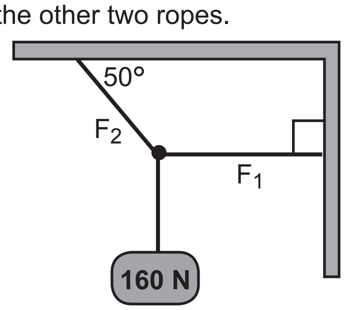
8. A 20 N painting hanging on a nail as shown in the figure, in such a way that the supporting rope is at an angle of 60°. What is the stress in each segment of the rope?



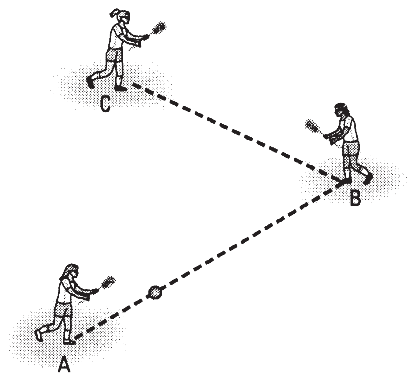
9. A 200 N ball hanging from a rope tied to two ropes, as shown in the figure. Find the stress in each of the ropes A, B and C.

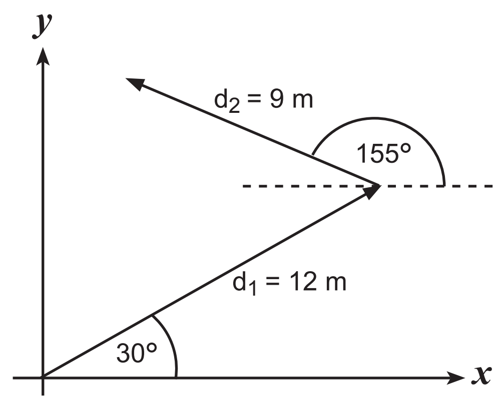


10. The other end of a rope to which a 160 N block is hung is tied at point O and supported by two other ropes as shown in the figure. Calculate the magnitude of the forces F_1 and F_2 exerted by the other two ropes.



11. Three tennis players skip a tennis ball to each other. Player a hits the ball a distance of 12 m to player B at an angle 30 º as in the figure below. Player B hits the ball to player C, 9 m away at an angle of 155 º. By the use of components find the ball's resultant displacement.





x-component	*y*-component
d_{1x} =	d_{1y} =
d_{2x} =	d_{2y} =
d_{Rx} (of Σd_x) =	d_{Ry} (of Σd_y) =