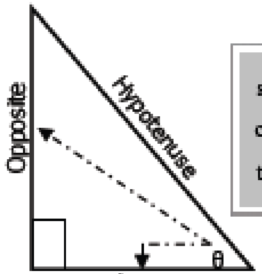
MONYETLA PROJECT SUMMER CAMP DEC 2020
MATHEMATICS
TRIGONOMETRY
GRADE 11
NOTES

TRIGONOMETRY

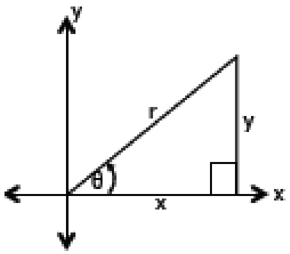
BASIC DEFINITIONS



$\sin\theta = \frac{o}{h}$ soh
$\cos\theta = \frac{a}{h}$ cah
$\tan\theta = \frac{o}{a}$ toa

These are our basic trig ratios.

On the Cartesian Plane



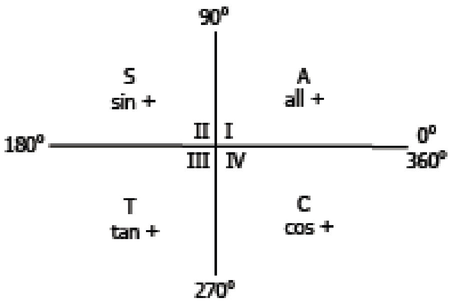
$\sin\theta = \frac{y}{r}$
$\cos\theta = \frac{x}{r}$
$\tan\theta = \frac{y}{x}$

Remember:

- $x^2 + y^2 = r^2$ (Pythagoras)
- Angles are measured upwards from the positive (+) x-axis (anti-clockwise) up to the hypotenuse (r).

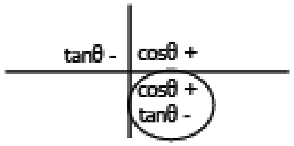
BASIC CAST DIAGRAM

Shows the quadrants where each trig ratio is +



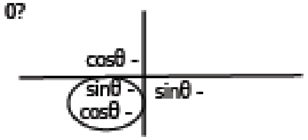
EXAMPLE

1. In which quadrant does θ lie if tanθ < 0 and cosθ > 0?



Quadrant IV

2. In which quadrant does θ lie if sinθ < 0 and cosθ < 0?



Quadrant III

FUNDAMENTAL TRIG IDENTITIES

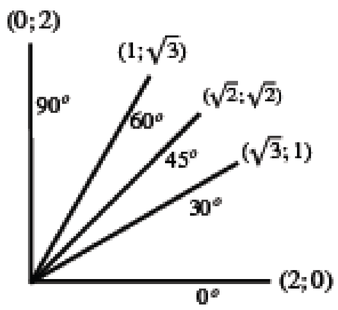
Memorise:

$$\frac{\sin A}{\cos A} = \tan A$$

$\sin^2 B + \cos^2 B = 1$
can be written as
$\sin^2 B = 1 - \cos^2 B$
$\cos^2 B = 1 - \sin^2 B$

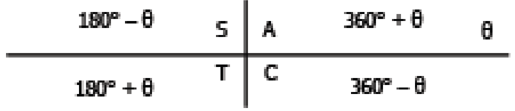
Special Angles

$r = 2$ $(x;y)$



REDUCTION FORMULAE

Reducing all angles to acute angles.



EXAMPLES

Reduce to an acute angle and simplify if possible (without a calculator):

1. $\sin 125^\circ$
$= \sin(180^\circ - 55^\circ)$
$= \sin 55^\circ$
(QII so sin is +)

2. $\cos 260^\circ$
$= \cos(180^\circ + 80^\circ)$
$= -\cos 80^\circ$
(QIII so cos is -)

3. $\tan 660^\circ$
$= \tan(360^\circ + 300^\circ)$
$= \tan 300^\circ$ (QI so tan is +)
$= \tan(360^\circ - 60^\circ)$
$= -\tan 60^\circ$ (QIV so tan is -)
$= -\frac{\sqrt{3}}{1}$
$= -\sqrt{3}$

Remember: 60° is a special angle

4. $\frac{\tan(180^\circ - \beta)\cos(180^\circ + \beta)\cos^2(360^\circ - \beta)}{\sin(360 + \beta)} + \sin^2(180^\circ + \beta)$
$= \frac{(-\tan\beta)(-\cos\beta)(\cos\beta)^2}{\sin\beta} + (-\sin\beta)^2$
$= \tan\beta . \frac{\cos^3\beta}{\sin\beta} + \sin^2\beta$
$= \frac{\sin\beta}{\cos\beta} . \frac{\cos^3\beta}{\sin\beta} + \sin^2\beta$
$= \cos^2\beta + \sin^2\beta$
$= 1$

Remember: Identities

Pythagoras Problems

Steps:
1. Isolate the trig ratio
2. Determine the quadrant
3. Draw a sketch and use Pythagoras
4. Answer the question

EXAMPLE

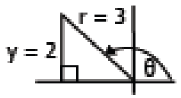
If $3\sin\theta - 2 = 0$ and $\tan\theta < 0$, determine $2\cos\theta + \frac{1}{\tan\theta}$ without using a calculator and using a diagram.

1. $3\sin\theta - 2 = 0$
$\sin\theta = \frac{2}{3}$ $\frac{y}{r}$

2.

$\therefore$ *Quadrant II*

3.



$x^2 + y^2 = r^2$
$x^2 + (2)^2 = (3)^2$
$x^2 = 5$
$x = \pm\sqrt{5}$
$\therefore x = -\sqrt{5}$

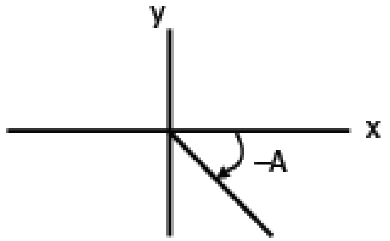
4. $2\cos\theta + \frac{1}{\tan\theta}$
$= 2(\frac{-\sqrt{5}}{3}) + \frac{1}{(\frac{2}{-\sqrt{5}})}$
$= \frac{-2\sqrt{5}}{3} - \frac{\sqrt{5}}{2}$
$= \frac{-4\sqrt{5} - 3\sqrt{5}}{6}$
$= \frac{-7\sqrt{5}}{6}$

Remember:
$\cos\theta = \frac{x}{r}$
and
$\tan\theta = \frac{y}{x}$

TRIGONOMETRY

NEGATIVE ANGLES

Angles measured downwards (clockwise) from the positive x-axis, which can be seen as Quadrant IV.



<u>Method 1: Q IV</u>

sin(–A) = –sinA
cos(–A) = cosA
tan(–A) = –tanA

<u>Method 2: Get rid of negative</u>

Add 360° to the angle to make it positive.

EXAMPLES

Simplify without the use of a calculator: sin(–330°)

NB: Negative Angle

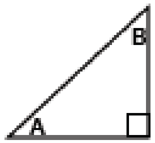
1) QIV

$$\begin{aligned}&\sin(-330^\circ)\\&= -\sin 330^\circ\\&= -\sin(360^\circ - 30^\circ)\\&= -(-\sin 30^\circ)\\&= \sin 30^\circ\\&= \frac{1}{2}\end{aligned}$$

2) $+360^\circ$

$$\begin{aligned}&= \sin(-330^\circ)\\&= \sin(360^\circ - 330^\circ)\\&= \sin 30^\circ\\&= \frac{1}{2}\end{aligned}$$

CO-FUNCTIONS



If A + B = 90° then sinA and cosB are known as co-functions.

sinA = sin(90° – B)
= cosB

EXAMPLES

1. $\sin 30^\circ = \sin(90^\circ - 60^\circ) = \cos 60^\circ$

2. $\cos 25^\circ = \cos(90^\circ - 65^\circ) = \sin 65^\circ$

> **NOTE:**
> Look at the quadrant first, **THEN** use the reduction/co-function formulae

3. $\sin(90^\circ - \alpha) = \cos\alpha$ — QI, so $\sin +$; $90^\circ \therefore \sin \leftrightarrow \cos$

4. $\cos(90^\circ + \beta) = -\sin\beta$ — QII, so $\cos -$; $90^\circ \therefore \sin \leftrightarrow \cos$

5. $\sin(\theta - 90^\circ) = -\cos\theta$ — QIV, so $\sin -$; $90^\circ \therefore \sin \leftrightarrow \cos$

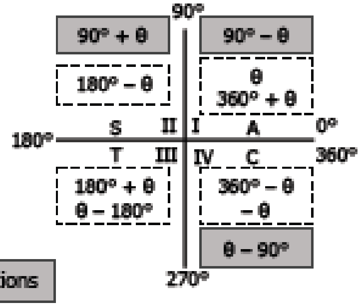
6. Simplify to a ratio of 10°:

a) $\cos 100^\circ = \cos(90^\circ + 10^\circ) = -\sin 10^\circ$ — QII, so $\cos -$; $90^\circ \therefore \sin \leftrightarrow \cos$

b) $\tan 170^\circ = \tan(180^\circ - 10^\circ) = -\tan 10^\circ$ — QII, so $\tan -$; $180^\circ \therefore$ *reduction*

FULL CAST DIAGRAM

Memorise the following diagram:



* Reductions | Co-functions

PROVING IDENTITIES

Steps:

1. Separate LHS and RHS
2. Start on the more complex side
3. Prove that the sides are equal.

EXAMPLES

1. $\cos^2 x \,.\, \tan^2 x = \sin^2 x$

$$\begin{aligned}\text{LHS} &= \cos^2 x \,.\, \tan^2 x\\&= \cos^2 x \,.\, \frac{\sin^2 x}{\cos^2 x}\\&= \sin^2 x = \text{RHS}\end{aligned}$$

2. $1 - 2\sin x \,.\, \cos x = (\sin x - \cos x)^2$

$$\begin{aligned}\text{RHS} &= (\sin x - \cos x)^2\\&= \sin^2 x + \cos^2 x - 2\sin x \,.\, \cos x\\&= 1 - 2\sin x \,.\, \cos x = \text{LHS}\end{aligned}$$

3. $\tan x + \dfrac{\cos x}{1 + \sin x} = \dfrac{1}{\cos x}$

$$\begin{aligned}\text{LHS} &= \tan x + \frac{\cos x}{1 + \sin x}\\&= \frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x}\\&= \frac{\sin x(1 + \sin x) + \cos x(\cos x)}{\cos x\,(1 + \sin x)}\\&= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)}\\&= \frac{\cancel{\sin x + 1}}{\cos x\cancel{(1 + \sin x)}}\\&= \frac{1}{\cos x} = \text{RHS}\end{aligned}$$

4. $\tan(155^\circ)$

$= \tan(180^\circ - 25^\circ)$ Q II, tan -
$= -\tan 25^\circ$

This can be solved in two ways:

Method 1: Ratio

$$= \frac{-\sin 25^\circ}{\cos 25^\circ} = \frac{-\sqrt{1 - p^2}}{p}$$

Method 2: Sketch

$$= \frac{-y}{x} = \frac{-\sqrt{1 - p^2}}{p}$$

PROBLEM SOLVING:

If cos25° = p, express the following in terms of p (i.e. get all angles to 25°):

1. $\cos(-385^\circ)$ negative angle, so a) + 360° or b) Q IV

$$\begin{aligned}&= \cos(-25^\circ)\\&= \cos 25^\circ\\&= p\end{aligned}$$

- 385 Q I
$\therefore$ + cos

2. $\sin(65^\circ)$

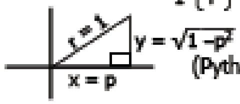
$= \sin(90^\circ - 25^\circ)$ Q I, sin +
$= \cos(25^\circ)$
$= p$

3. $\sin(335^\circ)$

$= \sin(360^\circ - 25^\circ)$ Q IV, sin -
$= -\sin 25^\circ$

REMEMBER: Correct angle is 25° BUT wrong sin ratio. Thus draw sketch.

Given $\cos 25^\circ = \dfrac{p}{1}\left(\dfrac{x}{r}\right)$



(Pythag)

So, $-\sin 25^\circ = \dfrac{-y}{r}$

$$= \frac{-\sqrt{1 - p^2}}{1} = -\sqrt{1 - p^2}$$